

**Math 101 – WORKSHEET 3**  
**THE DEFINITE INTEGRAL**

(1) (Sums) Given  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  find

- (a)  $\sum_{i=1}^{2n} i$
- (b)  $\sum_{i=1}^n (2i)$

(2) (Riemann sums)

- (a) Express the area between the  $x$ -axis, the lines  $x = 1$  and  $x = 4$  and the graph of  $f(x) = \cos(x^2)$  as a limit. Use the right-hand rule.

(b) Express  $\lim_{n \rightarrow \infty} \frac{1}{2n} \sum_{i=1}^n \tan\left(\frac{i}{3n}\right)$  as an integral and as an area.

*Remark.* For any choice of  $\Delta x$  (proportional to  $\frac{1}{n}$ ) and any choice of  $a$ , there is a solution, and they are all correct. The first choice is perhaps the most natural one, but there is no one single answer to this problem. Those who already know about “change of variables” in integrals can see how all the answers are related.

(3) Evaluate

(a)  $\int_0^7 4x dx$

(b)  $\int_{-1}^1 \sqrt{1-x^2} dx$

(c)  $\int_{-2}^2 (3+x) dx$