

**Math 101 – WORKSHEET 2**  
**AREA UNDER A CURVE**

- (1) Let  $A$  be the area lying between the  $x$ -axis, the curve  $y = x^2$  and the lines  $x = 0$ ,  $x = 1$ .  
(a) Draw a picture

- (b) Dividing the interval  $[0, 1]$  into two equal-width strips, show that  $A \leq \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{2} \cdot 1^2 = \frac{5}{8}$ .  
(c) Using the same subdivision, show that  $A \geq \frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{8}$ .

- (d) Using a subdivision into 3 strips, show  $\frac{1}{3} \cdot 0^2 + \frac{1}{3} \left(\frac{1}{3}\right)^2 + \frac{1}{3} \left(\frac{2}{3}\right)^2 \leq A \leq \frac{1}{3} \left(\frac{1}{3}\right)^2 + \frac{1}{3} \left(\frac{2}{3}\right)^2 + \frac{1}{3} \cdot 1^2$ .

- (e) For better accuracy, we use rectangles whose height is given by the function value at the *middle* of the strip. What do you get now?

(2) The  $\Sigma$  notation:

(a) Write an expression for the sum of the square roots of the integers between 5 and 32.

(b) How many terms are there in this sum?

(c) Write an expression for the sum of the integers between 1 and  $n$ .

(d) Write an expression for the sum of the integers between 1 and  $n$ .