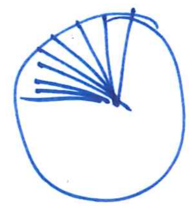
Math 101, lecture 2, 6/1/17

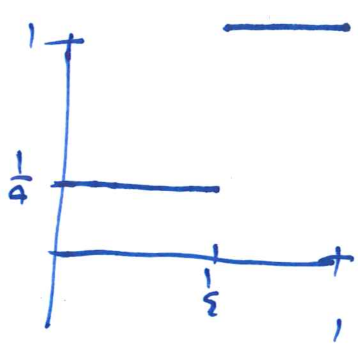
Goals: (1) Approximating areas
(2) Areas as limits
(3) Σ

Last time: "chop up and sum"



Summary: (1) chopped up disc into sectors ← approximate triangles
(2) Added up approximate areas
(3) took limit $n \to \infty$
number of triangles

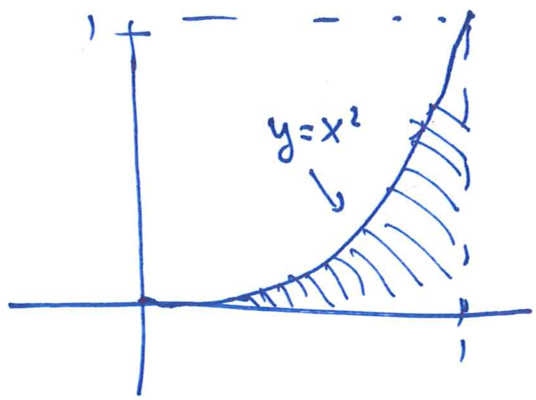
Example:



Math 101 – WORKSHEET 2
AREA UNDER A CURVE

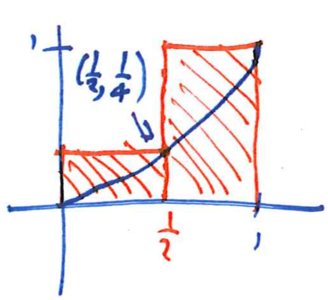
(1) Let A be the area lying between the x-axis, the curve $y = x^2$ and the lines $x = 0$, $x = 1$.

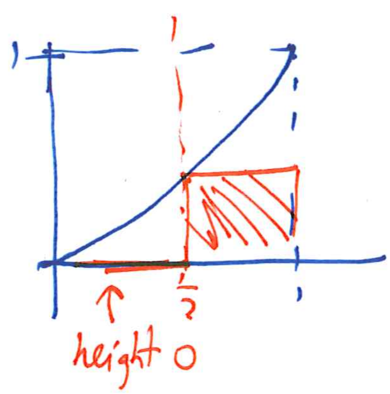
(a) Draw a picture



(b) Dividing the interval $[0, 1]$ into two equal-width strips, show that $A \leq \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{2} \cdot 1^2 = \frac{5}{8}$.

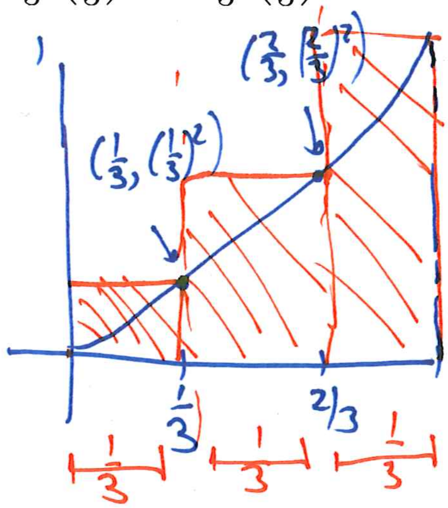
(c) Using the same subdivision, show that $A \geq \frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{8}$.





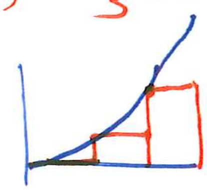
Date: 6/1/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(d) Using a subdivision into 3 strips, show $\frac{1}{3}\cdot 0^2 + \frac{1}{3}\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^2 \leq A \leq \frac{1}{3}\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^2 + \frac{1}{3}\cdot 1^2$.

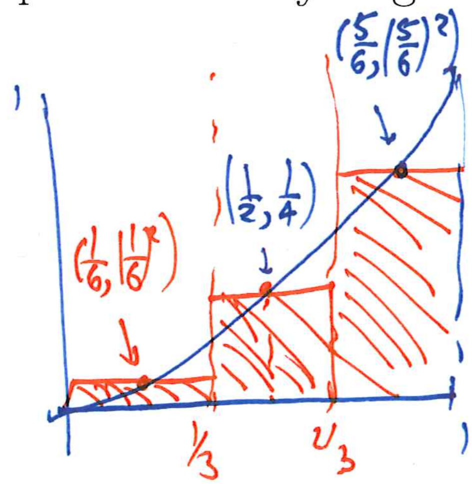


← over estimate area of rectangles is

$\frac{1}{3}\cdot\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)\cdot 1^2$



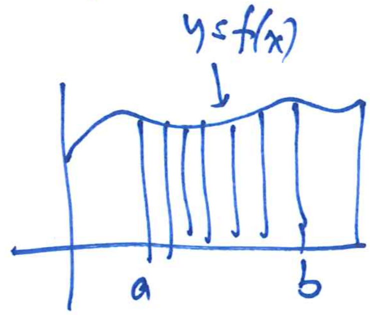
(e) For better accuracy, we use rectangles whose height is given by the function value at the *middle* of the strip. What do you get now?



$A \approx \frac{1}{3}\cdot\left(\frac{1}{6}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{5}{6}\right)^2$

Summary

To approximate area under graph of $y=f(x)$, between $x=a$, $x=b$.



(1) Divide interval into n pieces

(2) On each piece raise rectangle to a height $f(\cdot)$

$\cdot$ can be
- * left endpoint
- * right endpoint
- * midpoint
- * any other point.

(3) Add up the areas of rectangles

(4) Take <u>limit</u> as $n \to \infty$

Improve notation

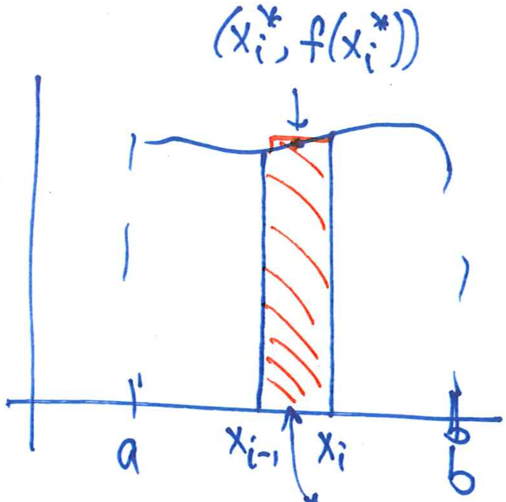
use index i to label intervals

divide $[a,b]$ into n subintervals, each has length $\Delta x = \frac{b-a}{n}$

first	interval	ends	at	$x_1 = a + \Delta x$
second	"	"	"	$x_2 = a + 2\Delta x$
ith	"	"	"	$x_i = a + i\Delta x$

ith interval is $[x_{i-1}, x_i]$

$$x_n = a + n\Delta x = a + n\frac{b-a}{n} = a + (b-a) = b$$



$x_i^* \leftarrow$ some point in $[x_{i-1}, x_i]$

Area of one rectangle is
$$\Delta x \cdot f(x_i^*)$$

Approximate area = sum over rectangles

$$= f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x$$

Possible choices of x_i^*:

(1) left-side rule $x_i^* = x_{i-1} = a + (i-1)\Delta x$

(2) right-side rule $x_i^* = x_i = a + i\Delta x$

(3) midpoint rule $x_i^* = \dfrac{x_i + x_{i-1}}{2} = a + (i - \frac{1}{2})\Delta x$

$\vdots$

"sum"

Notation: $\displaystyle\sum_{i=1}^{i=n} f(x_i^*)\Delta x \leftarrow$ "Riemann sum" for the area

Example: $f(x) = x^2$, $a = 0$, $b = 1$. $\Delta x = \dfrac{1}{n}$

Estimate area using right-hand rule. $x_i = 0 + \dfrac{i}{n} = \dfrac{i}{n}$

Area $\approx \displaystyle\sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n}$

~~Examples~~