Math 101 – WORKSHEET 21
SEPARABLE DIFFERENTIAL EQUATIONS

1. What is a DE?

(1) Consider the differential equation $y' = 3y^2$
   (a) For which values of C, D is $f(x) = Cx^D$ a solution to the equation?

   (b) Suppose $f(x)$ is a solution. Show that $f(x-a)$ is also a solution for any a. What is the solution with $f(0) = 1$?

2. Separation of variables

(2) Solve the follwing equations using separation of variables
   (a) $y' = x^3$

   (b) $y' = 5y$

   (c) (Final, 2012) $y' = xy$, $y(0) = e$.

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(3) (Final 2014) Find the solution of the DE $x\frac{\mathrm{d}y}{\mathrm{d}x} + y = y^2$ that satisfies $y(1) = -1$.

(4) A physical system satisfies the equation $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$. There m, k, E are constants (mass, spring constant, energy, respectively) and $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ is the velocity.

(a) Solve the equation to obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = v =$

(b) Suppose $m = k = 1$ and $E = \frac{1}{2}$. Integrate both sides of $\frac{\mathrm{d}x}{\sqrt{1-x^2}} = \mathrm{d}t$ and find a formula for $x = x(t)$.

(c) Solve the problem for general m, k, E.