Math 101 – WORKSHEET 15
INTEGRATION USING PARTIAL FRACTIONS

1. TAIL END OF TRIG SUBSTITUTION

(1) (105 Final, 2014 + 101 Final, 2009) Convert $\int \left(3 - 2x - x^2\right)^{-3/2} \mathrm{d}x$ to a trigonometric integral.

2. PARTIAL FRACTIONS: PRELIMINARIES

(1) (Polynomials)
    (a) Which of the following is irreducible? $x^2 + 7$, $x^2 - 7$, $2x^2 + 3x - 4$, $2x^2 + 3x + 4$.

    (b) Factor the polynomials $x^2 - 3x + 2$, $x^3 - 4x$.

(2) (Preliminaries 2) Evaluate
    (a) $\int \frac{\mathrm{d}x}{3x+4} =$

    (b) $\int \frac{\mathrm{d}x}{(3x+4)^3} =$

    (c) $\int \frac{8x}{4x^2 - 4x + 5} \mathrm{d}x = \int \frac{8x}{((2x-1)^2 + 4)} \mathrm{d}x =$

3. Partial fractions expansion

(1) Find A, B such that $\frac{5x+3}{(x+2)(2x-3)} = \frac{A}{x+2} + \frac{B}{2x-3}$:
- Clear denominators to get $5x + 3 =$
- (Method 1) Simplify and solve for A, B.

(2) Apply Method 2 to find A, B, C such that
$$\frac{6x^2-26x+26}{x^3-6x^2+11x-6} = \frac{6x^2-26x+26}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}.$$

(3) Now consider $\frac{8x-10}{4x^3-4x^2+5x} = \frac{8x-10}{x(4x^2-4x+5)} = \frac{A}{x} + \frac{Bx+C}{4x^2-4x+5}$
   (a) Find A using method 2

   (b) Calculate $\frac{8x-10}{x(4x^2-4x+5)} - \frac{A}{x}$ to find B, C.

(4) Finally consider $\frac{x^2}{(x+2)(2x-3)}$. Can we have A, B such that $x^2 = A(x+2) + B(2x-3)$?