[16] **4.** Let $f(x) = x\sqrt{3-x}$.

(a) (2 marks) Find the domain of $f(x)$.

| Answer |
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| |

(b) (4 marks) Determine the $x$-coordinates of the local maxima and minima (if any) and intervals where $f(x)$ is increasing or decreasing.

(c) (2 marks) Determine intervals where $f(x)$ is concave upwards or downwards, and the $x$-coordinates of inflection points (if any). You may use, without verifying it, the formula $f''(x) = (3x-12)(3-x)^{-3/2}/4$.

Question 4 continued on the next page...

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Question 4 continued

(d) (2 marks) There is a point at which the tangent line to the curve $y = f(x)$ is vertical. Find this point.

| Answer |
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(e) (2 marks) The graph of $y = f(x)$ has no asymptotes. However, there is a real number $a$ for which $\lim_{x \to -\infty} \frac{f(x)}{|x|^a} = -1$. Find the value of $a$.

| Answer |
| --- |
| |

(f) (4 marks) Sketch the graph of $y = f(x)$, showing the features given in items (a) to (d) above and giving the $(x, y)$ coordinates for all points occurring above and also all $x$-intercepts.

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[14] **4.** Let

$$f(x) = \begin{cases} \frac{4}{\pi}\tan^{-1} x, & \text{if } x \geq 1, \\ 2 - x^4, & \text{if } x < 1. \end{cases}$$

[*Note:* Another notation for $\tan^{-1}$ is arctan.]

(a) (3 marks) Show that $f(x)$ is continuous at $x = 1$.

(b) (1 mark) Determine the equations of any asymptotes (horizontal, vertical or slant).

(c) (4 marks) Determine all critical numbers, open intervals where $f$ is increasing or decreasing, and the $x$-coordinates of all local maxima or local minima (if any).

Question 4 continues on the next page...

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Question 4 continued

(d) (2 marks) Determine open intervals where the graph of $f$ is concave upwards or concave downwards, and the $x$-coordinates of all inflection points (if any).

(e) (4 marks) Sketch the curve $y = f(x)$, showing all the features given in items (a) to (d) above and giving the $(x, y)$ coordinates for all points occurring above (if any).

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