MATH 100 – WORKSHEET 16
THE MVT

1. More minima and maxima

(1) Show that the function $f(x) = 3x^3 + 2x - 1 + \sin x$ has no local maxima or minima. You may use that $f'(x) = 9x^2 + 2 + \cos x$.

(2) Let $g(x) = xe^{-x^2/8}$. Given that $g'(x) = \left(1 - \frac{x^2}{4}\right) e^{-x^2/8}$, find the absolute minimum and maximum of g on
  (a) $[0, \infty)$
  (b) $[-1, 4]$

(3) Find the critical numbers of $h(x) = \begin{cases} x^3 - 6x^2 + 3x & x \leq 3 \\ \sin(2\pi x) - 18 & x \geq 3 \end{cases}$.

2. The Mean Value Theorem

Theorem. *Let f be defined and differentiable on $[a, b]$. Then there is c between a, b such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.*
Equivalently, for any x there is c between a, x so that $f(x) = f(a) + f'(c)(x - a)$.

(1) Let $f(x) = e^x$ on the interval $[0, 1]$. Find all values of c so that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

(2) Let $f(x) = |x|$ on the interval $[-1, 2]$. Find all values of c so that $f'(c) = \frac{f(2)-f(-1)}{2-(-1)}$

(3) Show that $f(x) = 3x^3 + 2x - 1 + \sin x$ has exactly one real zero. (Hint: let a, b be zeroes of f. The MVT will find c such that $f'(c) = ?$)

(4) (Final 2012) If $f(1) = 3$, f is continuous on $[1, 4]$ and $f'(x) \leq -2$ for $x \in (1, 4)$, how large can $f(4)$ be?

(5) Show that $|\sin a - \sin b| \leq |a - b|$ for all a, b.

(6) Let $x > 0$. Show that $e^x > 1 + x$ and that $\ln(1 + x) \leq x$.