## MATH 100 - WORKSHEET 8 LOGARITHMS AND THEIR DERIVATIVES

## 1. Review of Logarithms

Logarithm rules.

$$
\log _{b}\left(b^{x}\right)=b^{\log _{b} x}=x
$$

$$
\log _{b}(x y)=\log _{b} x+\log _{b} y
$$

$$
\log _{b}\left(x^{y}\right)=y \log _{b} x
$$

$\log _{b} \frac{1}{x}=-\log _{b} x$
(1) $\log \left(e^{10}\right)=$

$$
\log \left(2^{100}\right)=
$$

(in terms of $\log 2$ )
(2) A variant on Moore's Law states that computing power doubles every 18 months. Suppose computers today can do $N_{0}$ operations per second.
(a) Write a formula for the power of computers $t$ years into the future:

- Computers $t$ years from now will be able to do $N(t)$ operations per second where

$$
N(t)=
$$

(b) A computing task would take 10 years for today's computers. Suppose we wait 3 years and then start the computation. When will we have the answer?
(c) At what time will computers be powerful enough to complete the task in 6 months?
2. Differentiation

$$
(\log x)^{\prime}=\frac{1}{x} \quad f^{\prime}=f \times(\log f)^{\prime}
$$

(1) Differentiate
(a) $f(t)=\log \left(t^{2}+3 t\right) \cdot f^{\prime}(t)=$
(b) $g(x)=x^{2} \log \left(1+x^{2}\right) \cdot g^{\prime}(x)=$
(c) $h(r)=\frac{1}{\log (2+\sin r)} \cdot h^{\prime}(r)=$
(d) Find $y^{\prime}$ if $\log (x+y)=e^{y}$
(2) Using the chain rule, $\frac{d(\log (a x))}{\mathrm{d} x}=$
(3) Use logarithmic differentiation to differentiate:
(a) $\frac{x^{5} \cos x}{\sqrt{5+x}}$
(b) $x^{x}$
(c) $(\log x)^{\cos x}$

