

**MATH 253 – WORKSHEET 31**  
**CYLINDRICAL COORDINATES**

Replace  $(x, y, z)$  with  $(r, \theta, z)$  by using polar coordinates in the  $xy$  plane:

$$\begin{aligned}x &= r \cos \theta & y &= r \sin \theta & z &= z \\r &= \sqrt{x^2 + y^2} & \tan \theta &= \frac{y}{x} & z &= z\end{aligned}$$

- (1) Express the following surfaces in cylindrical coordinates.
- (a) The cylinder of radius 2 about the  $z$ -axis.
  
  - (b) The paraboloid  $z = x^2 + y^2$ .
- (2) A drill bit of diameter  $a$  is used to drill a hole through a ball of radius  $a$ . What is the volume of the remaining object?

- (3) Where is the center of mass of a right circular cone? Suppose the base has radius  $R$  and the cone has height  $H$ .