MATH 100 – WORKSHEET 35
ANTIDERIVATIVES

1. Warmup

(1) Simple differentiation
 (a) Find one f such that $f'(x) = 1$.
  Solution: $\boxed{f(x) = x}$ works.
 (b) Find *all* such f.
  Solution: $\boxed{f(x) = x + c}$ is a solution for any constant c; these are all the solutions since if f is a solution, $(f(x) - x)' = 1 - 1 = 0$ so by the MVT $f(x) - x$ is constant.
 (c) Find f such that $f(7) = 3$.
  Solution: We have $f(x) = x+c$ for some c. Then $3 = f(7) = 7+c$ so $c = -4$ and $\boxed{f(x) = x - 4}$.

2. Antidifferentiation by massaging

(1) Find f such that $f'(x) = -\frac{1}{x}$.
 Solution: $\boxed{f(x) = -\ln|x|}$ works.

(2) Find f such that $f'(x) = \cos x.f'(x) = -\frac{1}{x}$.
 Solution: $\boxed{f(x) = \sin x}$ works

(3) Find all f such that $f'(x) = \cos x - \frac{1}{x}.f'(x) = -\frac{1}{x}$.
 Solution: By the sum rule, if $\boxed{f(x) = \sin x - \ln|x| + c}$ then $f'(x) = \cos x - \frac{1}{|x|}$ as desired.

(4) Find f such that $f'(x) = 2x^{1/3} - x^{-2/3}$ and $f(1000) = 5$.
 Solution: First recall that $\left(x^{4/3}\right)' = \frac{4}{3}x^{1/3}$ and $\left(x^{1/3}\right)' = \frac{1}{3}x^{-2/3}$. Now multiply by constants to get what we want: $\left(2 \cdot \frac{3}{4} \cdot x^{4/3}\right)' = 2x^{1/3}$ and $\left(3x^{1/3}\right)' = x^{-2/3}$ so f mus have the form $f(x) = \frac{3}{2}x^{4/3} - 3x^{1/3} + c$. The condition $f(1000) = 5$ then reads $\frac{3}{2}10^4 - 3 \cdot 10 + c = 5$ so $c = -14,695$ and $\boxed{f(x) = \frac{3}{2}x^{4/3} - 3x^{1/3} - 14,965}$.

(5) Find f such that $f''(x) = \sin x + \cos x$, $f(0) = 0$ and $f'(0) = 1$.
 Solution: Since $(f')' = f''$ we have $f'(x) = -\cos x + \sin x + c$ so $f(x) = -\sin x - \cos x + cx + d$. Note 2 constants since 2 anti-differentiations. Now $f'(0) = 1$ means $-\cos 0 + \sin 0 + c = 1$ so $c = 2$ and $f(0) = 0$ means $-\sin 0 - \cos 0 + 2 \cdot 0 + d = 0$ so $d = 1$ and finally $\boxed{f(x) = -\sin x - \cos x + 2x + 1}$.

(6) A cannonball is thrown off a tower of height H. Suppose that it starts from rest at the top of the tower and that its acceleration is constant (equal to g). When does it hit the ground?
 Solution: Put coordinates where vertical axis is y axis and the ball's position is $y(t)$,with velocity $v(t) = y'(t)$ and acceleration $a(t) = v'(t)$. Then $a(t) = -g$ (falling down) so $v(t) = -gt + v_0$ for some constant v_0. Setting $t = 0$ we see that $v_0 = 0$ (starting at rest) so $v(t) = -gt$. Now $y(t) = -\frac{g}{2}t^2 + y_0$ ($\left(t^2\right)' = 2t$, so now multiply by $-\frac{g}{2}$ to get $-gt$) and $y(0) = H$ so $y(t) = H - \frac{g}{2}t^2$. Finally, we need to find t such that $y(t) = 0$, that is $H - \frac{g}{2}t^2 = 0$ so $\boxed{t = \sqrt{\frac{2H}{g}}}$.

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