MATH 100 – WORKSHEET 30
L'HÔPITAL'S RULE

1. Statement

Theorem. *Let f, g be defined and differentiable on (a, b). Suppose that $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ and that $\lim_{x\to a} \frac{f'(x)}{g'(x)} = L$. Then $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists and equals L.*

Remark 1. The theorem also holds if $\lim_{x\to a} f(x), \lim_{x\to a} g(x)$ are both infinite in the extended sense, if $\lim \frac{f'(x)}{g'(x)}$ exists in the extended sense, and if we take $\lim_{x\to\infty}$.

(1) Evaluate $\lim_{x\to 1} \frac{\ln x}{x-1}$.

(2) Evaluate $\lim_{x\to 0} \frac{\cos x - 1}{x^2}$.

(3) Do (2) using a 2nd-order Taylor expansion.

(4) Given that $f(2) = 5$, $g(2) = 3$, $f'(2) = 7$ and $g'(2) = 4$ find $\lim_{x\to 3} \frac{f(2x-4)-g(x-1)}{g(x^2-7)}$.

(5) Evaluate $\lim_{x\to 0} \frac{e^x}{x}$.

(6) Evaluate $\lim_{x\to\infty} x^2 e^{-x}$.

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(7) Evaluate $\lim_{x\to 0} x \ln x$.

(8) Evaluate $\lim_{x\to\infty} x^n e^{-x}$.

(9) Suppose $a > 0$. Evaluate $\lim_{x\to\infty} x^{-a} \ln x$.

(10) Evaluate $\lim_{x\to 0}(2x+1)^{1/\sin x}$.