MATH 100 – WORKSHEET 15
LOGARITHMS AND THEIR DERIVATIVES

1. Logarithms

Summary.

$$\boxed{\log_b(b^x) = b^{\log_b x} = x}$$

$$\boxed{\log_b(xy) = \log_b x + \log_b y} \qquad \boxed{\log_b(x^y) = y\log_b x} \qquad \boxed{\log_b \frac{1}{x} = -\log_b x}$$

Review of calculations.

(1) Simplify the following logarithms
  (a) $\ln\left(e^{10}\right) =$
  (b) [Answer in terms of $\ln 2$]. $\ln(2^{100}) =$

(2) A drug in a patient has a metabolic half-life of 6 hours. Suppose a patient ingests a dose D_0 of the drug. Write a formula for the amount of drug present in the patient t hours afterward:

$$D(t) = D_0 \cdot 2^{-}$$

(3) A variant on Moore's Law states that computing power doubles every 18 months. Suppose computers today can do N_0 operations per second.
  (a) Write a formula for the power of computers t years into the future:
  - Computers t years from now will be able to do $N(t)$ operations per second where

$$N(t) =$$

  (b) A computing task would take 10 years for today's computers. Suppose we wait 3 years and then start the computation. When will we have the answer?
  (c) At what time will computers be powerful enough to complete the task in 6 months?

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2. Differentiation

$$\boxed{(\ln x)' = \frac{1}{x}} \qquad \boxed{f' = f\,(\ln f)'}$$

Example 1. Differentiate $\ln|x|$.

(1) Differentiate
  (a) $f(x) = x^2\ln(1+x^2)$. $f'(x) =$

  (b) $g(r) = \frac{1}{\ln(\sin r)}$. $g'(r) =$

  (c) $h(t) = \ln\left(t^2+3t\right)$. $h'(t) =$

  (d) Find y' if $\ln(x+y) = e^y$

(2) Logarithm Laws
  (a) Using the chain rule, $\frac{d(\ln(ax))}{\mathrm{d}x} =$

  (b) Simplify $\ln(ax)$ and explain why $(\ln(ax))' = (\ln x)'$. $\ln(ax) =$

(3) Differentiate using $f' = f\,(\ln f)'$.
  (a) $\frac{x\cos x}{\sqrt{5+x}}$

  (b) x^x

(c) $(\ln x)^{\cos x}$