1. A cylindrical tank with radius $5m$ is being filled with water at a rate of $3\text{m}^3/\text{min}$. How fast is the height of the water rising?
   Solution: Let R be the radius of the tank, $H(t)$ the height of the water at time t, and $V(t)$ the volume of the water. We then have $V(t) = (\pi R^2)H(t)$ and hence $V'(t) = (\pi R^2)\, H'(t)$ and $H'(t) = \frac{V'(t)}{\pi R^2}$. Plugging in the values we get

$$H'(t) = \frac{3}{\pi(5^2)}\frac{\text{m}}{\text{min}} = \frac{3}{25\pi}\frac{\text{m}}{\text{min}}\,.$$

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2. Two sides of a triangle have legnths $12m$ and $15m$. The angle between them is increasing at a rate of $2°/\text{min}$. How fast is the length of the third side increasing when the angle between the sides of fixed length is $60°$? For your reference, if the sides of the triangle are a, b, c and the angle α of the triangle is opposite the side of length a then the law of cosines reads

$$a^2 = b^2 + c^2 - 2bc\cos\alpha\,.$$

   Solution: Let b, c be the lengths of the sides of fixed length, α the angle between them, *measured in radians*. Let a be the length of the third side. Then differentiating the law of cosines we have

$$2a \cdot a' = 2bc\,(\sin\alpha)\,\alpha'$$

   and hence

$$a' = \frac{bc}{a}\,(\sin\alpha)\,\alpha'\,.$$

   Measuring side lengths in metres we plug in $b = 12$, $c = 15$, and at the time that $\alpha = 60°$ we have $a^2 = 12^2 + 15^2 - 2\cdot 12\cdot 15\cos 60° = 144 + 225 - 360\frac{1}{2} = 369 - 180 = 169$ so $a = \sqrt{169} = 13$. Next, $\sin 60° = \frac{\sqrt{3}}{2}$. Finally, at the time where $\alpha = 60°$, $\alpha' = 2°/\text{min} = 2\cdot\frac{2\pi}{360}\frac{\text{rad}}{\text{min}}$. It follows that

$$\begin{aligned} a' &= \frac{12\cdot 15}{13}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\pi}{90}\frac{\text{m}}{\text{min}} \\ &= \frac{\sqrt{3}\pi}{13}\frac{\text{m}}{\text{min}}\,. \end{aligned}$$

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3. Estimate $1001^{1/3} = \sqrt[3]{1001}$ using a linear approximation.
   Solution: Let $f(x) = x^{1/3}$. Then the linear approximation to f at $a = 1000$ is $f(x) = f(a) + f'(a)(x-a)$. Since $f'(x) = \frac{1}{3}x^{-2/3}$ we have $f(1000) = 10$, $f'(1000) = \frac{1}{3\cdot 100}$ so

$$f(1001) \approx 10 + \frac{1}{3\cdot 100}\cdot 1 = 10\frac{1}{300}\,.$$