1. Differentiate
$$(\tan x)^{1/x}.$$
Solution: Let $f(x) = (\tan x)^{1/x}$. Then $\ln f = \frac{1}{x}\ln(\tan x)$ and $(\ln f)' = -\frac{1}{x^2}\ln(\tan x) + \frac{1}{x\tan x}(1+\tan^2 x)$ so
$$f'(x) = f(x)\,(\ln f(x))' = -\frac{1}{x^2}(\tan x)^{\frac{1}{x}}\ln(\tan x) + \frac{1+\tan^2 x}{x}(\tan x)^{\frac{1}{x}-1}.$$

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2. A ball is thrown up. At time t it is at height $5t - 10t^2$. When is it at rest?
Solution: If the position is $y(t) = 5t - 10t^2$ then the velocity at time t is $v(t) = \frac{dy}{dt} = 5 - 20t$ and this is zero when $t = \frac{1}{4}$.

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3. A freshley brewed cup of coffee has temperature 95°C is a20°C room. After 30 minutes its temperature is 70°C. When it is cooling at the rate of 1°C/min?
Solution: We suppose that the temperature follows Newton's law of cooling, that is that for some constant k, $T' = k(T-20)$ where T is the temperature of the cup in degrees celsius. Switching to the variable $y = T - 20$ we have $y' = T'$ by the sum rule so that $y' = ky$. It follows that y decays exponentially: $y = Ce^{kt}$. If $t = 0$ is the time of the brewing then $y(0) = T(0) - 20 = 75$ so $C = 75$. At $t = 30$ minutes we have $y(30) = 75e^{30k}$ and also $y(30) = T(30) - 20 = 50$. It follows that
$$e^{30k} = \frac{50}{75} = \frac{2}{3}$$
so
$$k = \frac{1}{30}\ln\frac{2}{3}.$$
Finally, we need to find t such that $T' = -1$. Since $T' = y'$ and $y' = \left(Ce^{kt}\right)' = Cke^{kt}$ we need to find t such that
$$Cke^{kt} = -1$$
or
$$kt = \ln(-\frac{1}{Ck}) = -\ln(-Ck)$$
(note that k is negative!). It follows that the time is
$$t = -\frac{\ln(-Ck)}{k} = 30\frac{\ln(-Ck)}{\ln\frac{3}{2}} = 30\frac{\ln(\frac{75}{30}\ln\frac{3}{2})}{\ln(\frac{3}{2})} = 30\frac{\ln\left(\frac{5}{2}\right) + \ln\ln\left(\frac{3}{2}\right)}{\ln\left(\frac{3}{2}\right)}.$$