MATH 121 – EXERCIZE SET ON INTEGRALS
DUE IN CLASS ON WEDNESDAY, APRIL 4TH

1. Standard problems

1.1. **Calculate the following integrals. You will be primarily graded on the correctness of your results.**

(1) $\int_0^1 (x^3 - 2x + 5)\,\mathrm{d}x$
(2) $\int x\sqrt{x^2+a^2}\,\mathrm{d}x$
(3) $\int \frac{x}{\sqrt{x^2+a^2}} \cos\left(\sqrt{x^2+a^2}\right)\mathrm{d}x$
(4) $\int \frac{x}{\sqrt{1-x^4}}\,\mathrm{d}x$
(5) $\int_0^{\pi/2} e^x \cos x\,\mathrm{d}x$
(6) $\int e^{\sqrt{x}}\,\mathrm{d}x$
(7) $\int \frac{x+5}{x^3-2x^2+x}\,\mathrm{d}x$
(8) $\int x^3 \log x\,\mathrm{d}x$
(9) $\int \frac{\mathrm{d}x}{\left(x+\frac{1}{x}\right)\log(1+x^2)}$
(10) $\int \arctan x\,\mathrm{d}x$

2. Hyperbolic trig functions

The following subsitution technique is superior to trig substiutions for expressions of the form $\sqrt{x^2 \pm a^2}$.
Let $\cosh t = \frac{e^t+e^{-t}}{2}$, $\sinh t = \frac{e^t-e^{-t}}{2}$, $\tanh x = \frac{\sinh x}{\cosh x}$. The following points are not for submission.

- Verify for yourself that $\cosh t \geq 1$ for all t and that $\cosh t$ is an even function while $\sinh t$ is odd.
- Verify that $(\cosh t)' = \sinh t$ and that $(\sinh t)' = \cosh t$.
- Verify the key identity $\cosh^2 t - \sinh^2 t = 1$, that is $\cosh^2 t = 1 + \sinh^2 t$ and $\sinh^2 t = \cosh^2 t - 1$.
- Express the equation $x = \cosh t$. as a quadratic in e^t and verify that $t = \pm\log\left(x+\sqrt{x^2-1}\right) = \log\left(x \pm \sqrt{x^2-1}\right)$.
- Conclude similarly that $\operatorname{arcsinh} x = \log\left(x+\sqrt{x^2+1}\right)$.

Calculate the following integrals. You will be primarily graded on the correctness of your results.

(1) $\int \sqrt{1+x^2}\,\mathrm{d}x$
(2) $\int \frac{1}{\sqrt{4+x^2}}\,\mathrm{d}x$
(3) $\int x^2\sqrt{x^2-a^2}\,\mathrm{d}x$

3. A difficult challenge

Evaluate the following integral.

(1) $\int_0^\infty \frac{\log x}{1+x^2}\,\mathrm{d}x$.