Math 100 §105, Fall Term 2010
Sample midterm Exam

October 4th,2010

Student number:

LAST name:

First name:

Instructions

- Do not turn this page over. You will have 45 minutes for the exam.
- You may not use books, notes or electronic devices of any kind.
- Solutions should be written clearly, in complete English sentences, showing all your work.
- If you are using a result from the textbook, the lectures or the problem sets, state it properly.

Signature:

1	/18
2	/8
3	/6
4	/8
Total	/40

1 Short-form answers

Show your work and clearly delineate your final answer. Not all problems are of equal difficulty.

[3] a. Evaluate the following limit (or show it does not exist):

$$\lim_{x \to -2} \frac{x^2 - 4}{x^2 + 2x + 1}$$

[3] b. Evaluate the following limit (or show it does not exist):

$$\lim_{x \to 0} \frac{e^{3x} - 1}{x}$$

[3] c. Evaluate the following limit (or show it does not exist):

$$\lim_{x \to \infty} \frac{x \cos x}{x^2 + 1}$$

[3] d. Differentiate the following function:

$$\tan(e^{x/2})$$

[3] e. Given $f(1) = 1$, $f'(1) = 2$, $g(1) = 3$, $g'(1) = 4$ evaluate $h'(1)$ where

$$h(x) = \frac{xg^2(x)}{f(x)}\,.$$

[3] f. Evaluate the following limit (or show it does not exist):

$$\lim_{x\to 0} \frac{\sqrt{1-\cos x}}{x}$$

2 Long-form answers

[4] a. Let $f(x) = \frac{x}{x-1}$. Find $f'(x)$ *using the definition of the derivative.* No marks will be given for use of differentiation rules.

[4] b. Show that the equation $\cos x = x$ has a solution.

3 Long-form answers

A reaction occurs at the rate $R(x) = Ax^3 e^{-x/E}$ where x is the energy of the incoming particles, E is a constant energy scale, and A is an overall constant.

[4] a. Find the range of energies $x \geq 0$ for which a small increase in the energy will increase the rate of the reaction.

[2] b. Find the range of energies $x \geq 0$ for which a small increase in the energy will decrease the rate of the reaction. Your answers may depend on the constants A and E. Don't forget to justify them!

4 Long-form answers

[7] a. Write down two equations in the two unknowns a, b expressing the statement: "the line tangent to $y = \sqrt{x} - 1$ at the point where $x = a$ is also tangent to $y = x^2$ at the point where $x = b$".

[1] b. Solve the system of equations you have written down.