Nov 1

Laplace Tr. Cont'd

Miscellanious Matters.

What if initial cond's are not at time $t=0$? *

Example: $$w''(t) - 2w'(t) + 5w(t) = -8e^{\pi - t}$$

$$w(\pi) = 2 \qquad w'(\pi) = 12$$

Need to first transform the problem, e.g. define $\boxed{y(t) = w(t+\pi)}$

then $y'(t) = w'(t+\pi)$

$y''(t) = w''(t+\pi)$

$y(0) = w(\pi) = 2$

$y'(0) = w'(\pi) = 12$

Then replacing $t \to t+\pi$ in ODE: $w''(t+\pi) - 2w'(t+\pi) + 5w(t+\pi)$

$= -8e^{\pi-(t+\pi)}$

$= -8e^{-t}$

or in terms of y:

$$\begin{cases} y''(t) - 2y'(t) + 5y(t) = -8e^{-t} \\ \qquad y(0) = 2 \quad y'(0) = 12 \end{cases}$$

We can now apply the usual Laplace Tr. method (left as an exercise). We find that

$$y(t) = 3e^{t}\cos(2t) + 4e^{t}\sin(2t) - e^{-t}$$

To then find $w(t)$, use $w(t+\pi) = y(t) \Rightarrow y(t-\pi) = w(t)$

So

$$w(t) = y(t)\Big|_{t \to t-\pi} = \boxed{3e^{t-\pi}\cos\big(2(t-\pi)\big) + 4e^{t-\pi}\sin\big(2(t-\pi)\big) - e^{-(t-\pi)}}$$

* Thanks to Jessica Conway for this nice example