Midterm Test   Nov 10, 2010   Student Number: _____

Instructions: There are **6 pages** in this test (including this cover page).

1. **Caution: There may (or may not) be more than one version of this test paper.**

2. Ensure that your *full* name and student number appears on this page. Circle your section number.

3. No calculators, books, notes, or electronic devices of any kind are permitted.

4. Show all your work. Answers not supported by calculations or reasoning will not receive credit. Messy work will not be graded.

5. Five minutes before the end of the test period you will be given a verbal notice. After that time, you must remain seated until all test papers have been collected.

6. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. Continuing to write past this instruction will be considered dishonest behaviour.

7. Please remain seated and pass your test paper down the row to the nearest indicated aisle. Once all the test papers have been collected, you are free to leave.

8. Exposing your test paper, copying from another student's paper, or sharing information about this test constitutes academic dishonesty. Such behaviour may jeopardize your grade on this test, in this course, and your standing at this university.

9. There is a table of Laplace Transforms on p 6 of this test paper.

Question number	Grade	Value
1		18
2		10
3		16
4		16
Total		60

I have read and understood the instructions and agree to abide by them.

Signed: _____

Problem 1: Multiple Choice Questions: Circle ONE correct answer (a, b, c, d, or e). There is no partial credit in this question. Illegible or multiple answers will get no credit.

NOTE: In these questions, the notation for step functions includes $u_c(t) = H(t - c)$.

1: The Laplace transform of the function $f(t) = \dfrac{1}{t - 2}$ is

(a) e^{-2s}, (b) e^{2s}, (c) $u_2(s)e^{-2s}$, (d) 1, (e) None of these.

2 Which of the below corresponds to the inverse Laplace transform of $F(s) = \frac{1}{s-3}e^{-s}$

(a) $H(t - 1)e^{t-1}$, (b) $H(t - 3)e^t$, (c) $H(t - 3)e^{-t}$, (d) $H(t - 1)e^{3(t-1)}$, (e) $H(t - 3)e^{t-3}$

3 For the differential equation $y'' + 2y' + y = \delta(t)$ with initial conditions $y(0) = 0, y'(0) = 0$, we find that $F(s) = \mathcal{L}\{y(t)\}$ is which of the following functions?

(a) $\int_0^t y(t - \tau)\delta(\tau)d\tau$, (b) $\dfrac{1}{s^2 + s + 2}$, (c) $\dfrac{s}{s^2 + 2s + s}$, (d) $\dfrac{e^s}{s^2 + 2s + s}$, (e) $\dfrac{1}{(s + 1)^2}$

4: A "pirate-ship" ride at an amusement park is driven by a motor so that the vertical displacement of the "ship" satisfies $\alpha z'' + \beta z' + \frac{1}{\gamma}z = 10\cos(\omega t)$, where α, β, γ are manufacturer's specifications for the ride, and ω is the driving frequency of the motor powering the ride. Suppose that $\alpha = 28, \gamma = 7$ are fixed. For which of the following settings will the displacement of the ride have the greatest amplitude?

(a) $\beta = 0.05, \omega = 1/14$, (b) $\beta = 0.05, \omega = 4$, (c) $\beta = 2, \omega = \sqrt{3}/28$, (d) $\beta = 2, \omega = 1/2$, (e) $\beta = 4, \omega = 1/14$

5: Consider the system of first order ODEs given below.

$$\frac{dx}{dt} = x - 2y,$$
$$\frac{dy}{dt} = x + 3y.$$

Which of these answers best described the behaviour of the system?
(a) Growing exponential behaviour
(b) Decaying exponential behaviour
(c) Combined growing and decaying exponentials
(d) Oscillations with growing amplitude
(e) Oscillations with decaying amplitude

6: Consider the differential equation $y'' + 16y = E_0 \cos(\omega t)$. The following behaviour is observed when the driving frequency is $\omega = 4.4$. What is the frequency (radians/time) of the envelope of the oscillations and of the oscillations themselves?
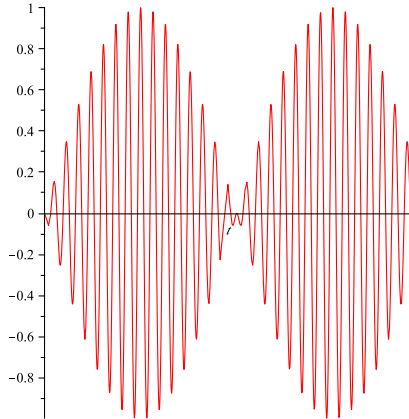
(a) Envelope frequency= 8.4, oscillation frequency 0.4
(b) Envelope frequency= 0.4, oscillation frequency 8.4
(c) Envelope frequency= 16/4.4, oscillation frequency 16·4.4
(d) Envelope frequency= 0.2, oscillation frequency 4.2
(e) Envelope frequency= 4.2, oscillation frequency 0.2

Problem 2: Consider the function shown in the figure. Answer (i) and (ii).

(i) Find the Laplace Transform of this function.
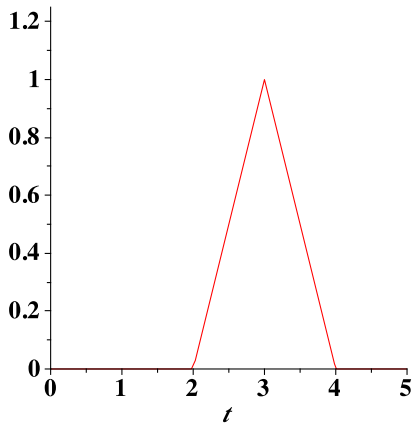
$\mathcal{L}\{f(t)\} =$ _____

(ii) If the same function is extended so that it is periodic, with period $T = 5$, what would be the Laplace transform of this new function?

$\mathcal{L}\{f_p(t)\} =$ _____

Problem 3: Short Answer Questions
Check your answer carefully, as there are no part marks for right method(s) in this question.

(A) Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{9}{s^5}$

$\mathcal{L}^{-1}\{F(s)\} = $ _____

(b) $F(s) = \dfrac{2}{(s+2)(s+1)}$

$\mathcal{L}^{-1}\{F(s)\} = $ _____

(c) $F(s) = \dfrac{s}{(s^2 + 2s + 7)}$

$\mathcal{L}^{-1}\{F(s)\} = $ _____

(B) Consider the ODE
$$y'' + 9y = t\cos(3t)$$

Using the Method of Undetermined Coefficients, what would be the form of the particular solution to this equation?

NOTE: Do not solve the equation and do not find the coefficients.

$y_p(t) = $ _____

Problem 4:

In a frictionless spring-mass system, the mass is $m = 2$kg and the spring constant is $k = 2$m kg s^{-2}. The vertical displacement of the spring satisfies

$$m\frac{d^2y}{dt^2} + ky = F(t), \quad y(0) = 0, \, y'(0) = 0. \tag{1}$$

(a) At $t = 0$, the mass is hit with a hammer, producing a unit impulse. You may assume that this impulse is represented by the Dirac delta function. Solve Eqn. (1) to find the displacement $y(t)$ at $t > 0$.

$y(t) = $ _____

(b) What would be the velocity of the mass, $v(t)$ in the same system with the same applied force?

$v(t) = $ _____

(c) Consider the functions $g(t) = t$ and $h(t) = \sin(t)$. Compute the convolution $g * h$.

[Hint: A useful fact is: $\int x \sin(x) dx = \sin(x) - x \cos(x)$.]

$g * h = $ _____

Problem 4 is continued next page → .. →

Problem 4 Cont'd

(d) In a new experiment, a time dependent force of the form $g(t) = t$ is applied to the spring-mass system. Use the Laplace transform method to solve Eqn. (1) with this time-dependent force. (You may find the result in (c) useful.)

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$\delta(t-c)$	e^{-cs}
$e^{ct}f(t)$	$F(s-c)$
$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$