In-class problem:

$$y'' + 4y = 5\cos(3t) \qquad y(0)=0,\ y'(0)=0$$

- Corresp. hom. problem $\quad y''+4y=0$

  char. eqn $\quad r^2+4=0 \qquad$ roots $\quad r^2=-4$

  $$r = \pm i\sqrt{4} = \pm \underset{\substack{\uparrow \\ \text{"natural frequency"}}}{2}i$$

  soln to hom problem:

  $$y(t) = C_1\cos(2t) + C_2\sin(2t)$$

- Particular soln : guess :

  $$Y_p(t) = \underbrace{A\cos(3t)}_{\text{form of forcing function}} + \underbrace{B\sin(3t)}_{\text{may need this term too}}$$

  This does not duplicate the solns to hom problem (since frequencies not same) so guess should be fine.

- Find A, B: need

  $$Y_p'(t) = -3A\sin(3t) + 3B\cos(3t)$$

  $$Y_p''(t) = -9A\cos(3t) - 9B\sin(3t)$$

  plug into $y''+4y = 5\cos(3t)$:

$$\left[-9A\cos(3t) - 9B\sin(3t)\right] + 4\left[A\cos(3t) + B\sin(3t)\right] = 5\cos(3t)$$

sort the terms:

$$\cos(3t)\left[-9A+4A\right] = 5\cos(3t) \quad \Rightarrow \quad -5A = 5 \qquad A=-1$$

$$\sin(3t)\left[-9B+4B\right] = 0 \quad \Rightarrow \quad B=0$$

so

$$Y_p(t) = -1\cdot\cos(3t) + 0\cdot\sin(3t) = \underbrace{-\cos(3t)}_{\text{partic. soln}}$$

- Gen'l soln:

  $$y(t) = \overbrace{C_1\cos(2t) + C_2\sin(2t)}^{\text{soln to hom. pr}} \overbrace{-\cos(3t)}^{\text{partic. soln}}$$

- Use initial cond's $\quad y(0)=0 \quad \Longrightarrow \quad C_1 - 1 = 0 \qquad C_1 = 1$

  $$y'(0)=0 \quad \Longrightarrow \quad C_2 = 0$$

- Soln: $\quad y(t) = \cos(2t) - \cos(3t)$

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