Course 2
Mathematical Analysis of Pattern Formation in Reaction-Diffusion System

Description: This is a continuation of Professor Michael Ward's lecture. In this part, I shall introduce mathematical tools developed in order to give a rigorous footing on the existence and stability of multiple spikes in reaction-diffusion systems. I will concentrate exclusively on the Gierer-Meinhardt system

$$(GM) \quad \begin{cases} u_t = \epsilon^2 \Delta u - u + \frac{u^p}{v^q}, & x \in \Omega, t > 0 \\ \tau v_t = D\Delta v - v + \frac{u^r}{v^s}, & x \in \Omega, t > 0 \end{cases}$$

and its shadow system.

One of the fundamental difficulty in studying reaction-diffusion system is the lack of variational structure. This forbids the use of powerful variational/energy method in elliptic equations. In this course, I will introduce two methods—one is Nonlocal Eigenvalue Problem (NLEP) method, and the other is the Finite/Infinite dimensional Liapunov-Schmidt reduction method.

Topic prerequisites: Basic PDE theory.

Textbook : Juncheng Wei and Matthias Winter, Mathematical Aspects of Pattern Formation in Biological Systems Applied Mathematical Sciences Series, Vol. 189, Springer 2014 , ISBN: 978-4471-5525-6.

Topics

- Introduction–Shadow Systems
- Study of Profile Function and Spectrum Theory
- Nonlocal Eigenvalue Problems (NLEP) method
- Introduction to finite-dimensional reduction method I: one-dimensional scalar equation

- Introduction to finite-dimensional reduction method II: higher-dimensional scalar equation
- Introduction to finite-dimensional reduction method III: one-dimensional Gierer-Meinhardt system
- Introduction to finite-dimensional reduction method IV: two-dimensional Gierer-Meinhardt system
- If time permits: Introduction to infinite-dimensional reduction method V: two-dimensional inhomogeneous Allen-Cahn equation

Final Remark: Any questions? Please send me an email or drop by my office Chase Building 315.