Homework 1

Math 615

February 22, 2024

Problem 1, simple manifolds with trivial canonical class.

Let M be a projective manifold and let $E \to M$ be a (holomorphic) vector bundle.

1. Suppose that $\Lambda^{rkE} E \cong K_M$. Prove that $X = \mathrm{Tot}(E)$, the total space of the vector bundle, is Calabi-Yau (in the sense that $K_X \cong \mathcal{O}_X$).
2. Suppose that $s : M \to E$ is a section which is transverse to the zero section so that $X = s^{-1}(0) \subset M$ is a submanifold. Find a condition on E so that X is Calabi-Yau (in the sense that $K_X \cong \mathcal{O}_X$).

Problem 2, the geometry of $\overline{\mathcal{M}}_2(\mathbb{P}^1, [\mathbb{P}^1])$.

Show that the moduli space $\overline{\mathcal{M}}_2(\mathbb{P}^1, [\mathbb{P}^1])$ has two components of dimension 4 and 5 respectively which meet each other in a space of dimension 3. Descibe each component and their intersection explicitly.

Problem 3, the geometry of $\overline{\mathcal{M}}_1(\mathbb{P}^1, 2[\mathbb{P}^1])$.

The moduli space $\overline{\mathcal{M}}_1(\mathbb{P}^1, 2[\mathbb{P}^1])$ has a stratification by the topological type of the domain curves of the stable maps. Describe all possible topological types of stable maps: make a table where each row in the table corresponds to a stratum with a fixed topological type and the columns of the table give

1. the dual graph of the map with vertex v_i labelled by (g_i, d_i), the genus and degree of the component corresponding to v_i,
2. the dimension of the stratum, and
3. a picture of the corresponding map.