Math 253 2017: Homework 2

1. **Spilled your cup.**

   (a) Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane.

   (b) Find a point on the surface where $y = 2$ and $z = 3$.

   (c) Identify the surface (by name).

   (d) Sketch a contour plot (in the usual xy-plane) of this surface for $z = 0, 1, 2, 3, 4$.

2. **Get b-b-back in my glass this m-m-minute.** Professor Calculus's rocket follows the path $\vec{r}(t) = \langle e^{-(t-4)}, 2t, t \rangle$. At time $t = 4$, Captain Haddock steps gently off the rocket to "go home to Marlinspike".

   (a) Give an equation for the path he follows, assuming no gravitation influence from other bodies. Give your answer as a function of time $\vec{l}(t)$. Hint: at time $t = 4$, his position must match that of the rocket.[1]

   (b) Show explicitly that Captain Haddock's position and velocity agrees with the rocket at $t = 4$.

[1] Orbital mechanics a bit rusty? Do *not* brush up by playing *Kerbal Space Program*; just assume he follows a straight line.

(c) Marlinspike Hall follows the path $\vec{m}(t) = \langle 5 - t, t, t^2 - 18t \rangle$. What is Captain Haddock's closest approach to Marlinspike? Hint: consider $f(t)$ to be the square of the distance between Captain Haddock and Marlinspike.

(d) At what time does his closest approach occur? Give your answer as an approximate decimal value to at least four significant digits.

3. **l'Hôpital de l'Aventure.** In which Jack, Lucy, Dinah and Philip try to find a mysterious hospital rumoured to be found at $(0, 0, \frac{1}{2})$. Let

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Consider

$$\lim_{(x,y)\to(0,0)} f(x,y).$$

(a) Compute the limit along the lines $y = mx$, where $m \in \mathbb{R}$, $m \neq 0$.

(b) Compute the limit along the path $y = x^2$.

(c) Is this function continuous at the origin? Explain briefly.

(d) Suppose P is a point which is not the origin. Is this function continuous at P? Explain in words; no calculations are necessary.

(e) Using software of your choice, make a *nice looking* plot the surface $z = f(x, y)$. Attach your plot to your homework.

4. **The 17-page exam.** Complete the attached exam problem. There is an easy way to compute the normals here which we have not learned yet, so that information is filled in for you.

OLD EXAM

[3 marks] 1. (a) Find the equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 9$ at the point $(2, 2, 1)$ on that surface.

Hint: the normal vector to the surface at that point is $\vec{n} = \langle 4, 4, 2 \rangle$ (we will learn an easy way to find this later).

Answer:

[2 marks] (b) Find the equation of the tangent plane to the surface $x^2 + y^2 - 8z^2 = 0$ at the point $(2, 2, 1)$ on that surface.

Here the normal is $\vec{n} = \langle 4, 4, -16 \rangle$.

Answer:

3 marks (c) The planes of parts (a) and (b) intersect in a line which passes through the point $(2, 2, 1)$. Write an equation for that line of intersection, in parametric form.

Answer:

2 marks (d) Find the points on the line of part (c) which are distance 1 from $(2, 2, 1)$.

Answer: