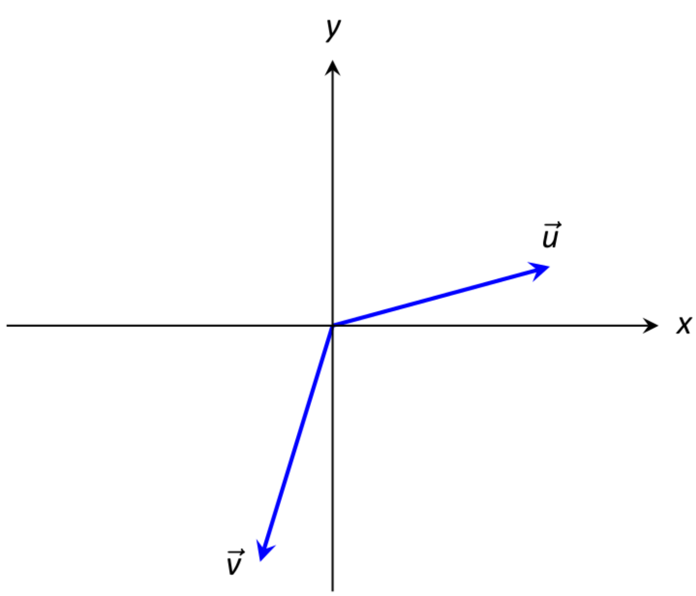
Math 253 HW1 2017 T1

You can draw some of these figures on the HW sheet itself. If you need more space, attach any additional written work.

Q1 Parallelograms in $\mathbb{R}^3$. (a) APEX 10.1 #8. (b) Also sketch and carefully label this object on a "cavalier oblique axis" (like the ones we use in lectures).

(c) Where is the center of the shape?

Q2 Vector arithmetic. [From APEX 10.2 #12.] Sketch the vectors $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$ and $2\vec{u}-\vec{v}$ on the same axes.



Q3 Romanticizing rejection. "There's nothing romantic about rejection. It's horrible." – Marlon James

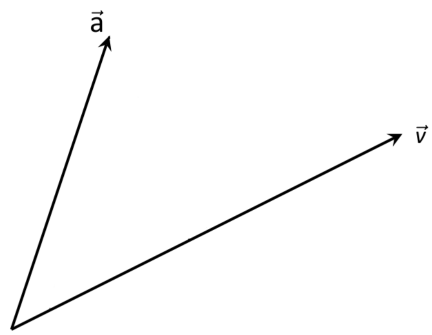
(a) In terms of the dot product, what is the vector projection of a vector $\vec{a}$ onto a vector $\vec{v}$?

$$\text{proj}_{\vec{v}}\vec{a} =$$

Now define the "*vector rejection*" of $\vec{a}$ onto $\vec{v}$ by

$$\vec{w} = \vec{a} - \text{proj}_{\vec{v}}\vec{a} =$$

(b) Sketch and label $\text{proj}_{\vec{v}}\vec{a}$ and $\vec{w}$ on the following figure:



(c) Show (prove) that $\vec{w}$ is orthogonal to $\vec{v}$.

(c) In two dimensions, now suppose $\vec{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Consider a triangle formed by the points $A = (10, 20)$, $B = (20, 10)$ and $C = (0, 10)$. By considering each point as a vector a from the origin, compute the vector rejection onto v. Plot the resulting three points.

Fig for (c):

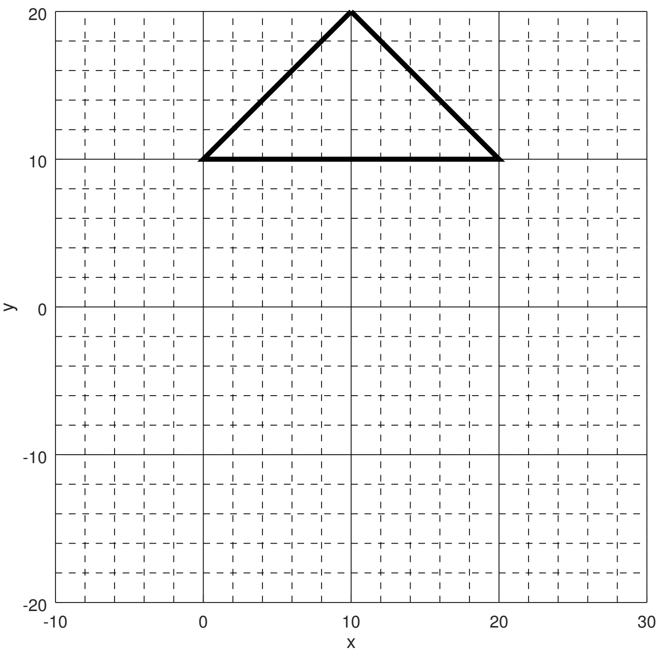
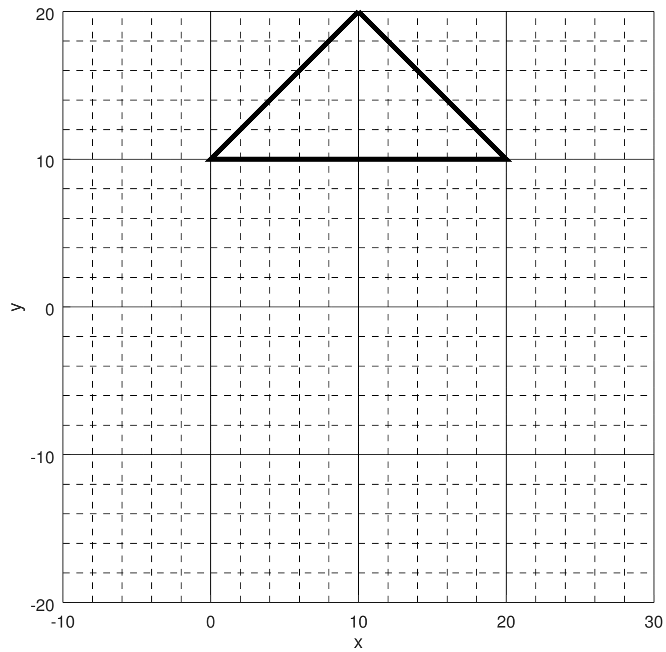


Fig for (e):



(d) Can you make a conjecture about what the vector rejection onto v does to *any* point in $\mathbb{R}^2$?

(e) Try computing $\vec{z} = \vec{a} - 2\text{proj}_{\vec{v}}\vec{a}$ for each of the vertices of the triangle (again, taking these as $\vec{a}$). Plot those points. What does this new operation do to the triangle? Can you think of a good name for it?

Q4 Naughts and Crosses. [Adapted from the Stewart textbook.]

(a) Let $\vec{u} = \langle 1, 2, 1 \rangle$. Find all vectors $\vec{v}$ such that

$$\vec{u} \times \vec{v} = \langle 3, 1, -5 \rangle.$$

(b) Of all the answers to (a), which one has no component in the **i** direction?

(c) Are there any vectors $\vec{w}$ such that $\vec{u} \times \vec{w} = \langle 3, 1, 5 \rangle$? Justify your answer.