

NEW FOREST EXAMPLE

board 1 foot square and 1 inch thick; close equivalent American measure is the board foot.) The management of New Forest had to choose a felling program for an area of about 30,000 acres, with the objective of maximizing the net discounted revenue over the next decade. The problem considered here involves only part of that area; some 8,500 acres with six different crop types shown in Table 11.1.

Table 11.1 New Forest Crop Types

Crop type	Description	Acres	Volume if felled (h.ft./acre)
1	High-volume hardwoods	2,754	2,000
2	Medium-volume hardwoods	850	1,200
3	Low-volume hardwoods	855	700
4	Conifer high forest	1,598	4,000
5	Mixed high forest	405	2,500
6	Bare land	1,761	

The hardwood areas are further classified into those with a complete undergrowth, those with a partial undergrowth, and those with no undergrowth. The corresponding acreages are shown in Table 11.2.

Table 11.2 Classification of Hardwood Areas

	Complete undergrowth	Partial undergrowth	No undergrowth	Total
High-volume hardwoods	357	500	1,897	2,754
Medium-volume hardwoods	197	130	523	850
Low-volume hardwoods	39	170	646	855

Any number of acres of any crop type can receive one of two basic treatments: fell and plant conifer (treatment 1A) or fell and plant hardwood (treatment 1B). When applied to bare land, these treatments become "plant conifer" or "plant hardwood." In addition, for hardwood areas with a complete undergrowth, management has the option of felling and retaining the undergrowth (treatment 2); similarly, for hardwood areas with a partial undergrowth, management has the option of felling and enriching the undergrowth (treatment 3). A final option is simply to postpone treatment altogether for any number of acres of any crop type.

The net discounted revenue (NDR) over the next ten years varies with treatment and crop type. These figures, in pounds per acre (£/acre), are estimated in Table 11.3.

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Table 11.3 Estimate of Net Discounted Revenue (£/acre)

Crop type	Treatment			No treatment
	1A	1B	2	
1	287	215	228	292
2	207	135	148	212
3	157	85	98	162
4	487	415	—	—
5	337	265	—	—
6	87	15	—	—

In abstract jargon, the various treatments would be referred to as activities and the various areas with different crop types would be referred to as resources; hence the title of the present chapter.

Visual amenity requirements and a limited labor capacity dictate the following four conditions.

- (i) The treated area must not exceed 5,000 acres.
- (ii) The resulting conifer area must not exceed 3,845 acres.
- (iii) The volume of felled hardwood must not exceed 2.44 million h.ft.
- (iv) The volume of felled conifer and mixed high forest must not exceed 4.16 million h.ft.

The conifer area in (iii) is the area of newly planted conifer plus the untreated area of old conifer. Estimates of the average volume per acre of each of the five crops are listed in Table 11.1.

To formulate the problem in linear programming terms, let us first examine Table 11.3. For each of the six crop types, the NDR resulting from treatment 1A exceeds the NDR resulting from treatment 1B by £72. This is hardly surprising: the NDR brought in by felling and planting equals the sum of two components, the NDR brought in by felling plus the NDR brought in by planting. Actually, the first component (the NDR brought in by felling) comes to £1 per 10 h.ft. felled; the second component (the NDR brought in by planting) is £87 per acre if conifer is planted, and £15 per acre if hardwood is planted. Thus, where only felling is concerned, the distinction between treatments 1A and 1B is irrelevant; we may refer to both 1A and 1B as treatment 1. On the other hand, where only planting is concerned, the crop types formerly occupying the area are irrelevant: all that matters are the total acreages of newly planted conifer and newly planted hardwood.

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max 204x10+287x11a+215x11b+228x12+292x13
+148x20+201x21a+135x21b+148x22+212x23
+112x30+157x31a+85x31b+98x32+162x33
+371x40+487x41a+415x41b
+264x50+337x51a+265x51b
+61x60+87x61a+15x61b
subject to
hivolhd)x10+x11a+x11b+x12+x13=2754
mdvolhd)x20+x21a+x21b+x22+x23=850
lovolhd)x30+x31a+x31b+x32+x33=850
conifhi)x40+x41a+x41b=1598
mixedhi)x50+x51a+x51b=405
barelnd)x60+x61a+x61b=1761
conifer)x11a+x21a+x31a+x41a+x51a+x61a+x40<3845
treatmnt)x11a+x11b+x12+x13+x21a+x21b+x22+x23+x31a+x31b+x32+x33
+x41a+x41b+x51a+x51b+x61a+x61b<5000
felledhd)2000x11a+2000x11b+2000x12+2000x13
+1200x21a+1200x21b+1200x22+1200x23
+700x31a+700x31b+700x32+700x33<2440000
felledot)4000x41a+4000x41b+2500x51a+2500x51b<4160000
x12<357
x22<197
x32<39
x13<500
x23<130
x33<170
x11b+x21b+x31b+x41b+x51b+x61b>500
end

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LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

1) 1839510.

VARIABLE	VALUE	REDUCED COST
X10	1888.769287	0.000000
X11A	365.230774	0.000000
X11B	0.000000	0.000000
X12	0.000000	34.461540
X13	500.000000	0.000000
X20	720.000000	0.000000
X21A	0.000000	6.615385
X21B	0.000000	0.615385
X22	0.000000	35.076923
X23	130.000000	0.000000
X30	59.230770	0.000000
X31A	120.769234	0.000000
X31B	500.000000	0.000000
X32	0.000000	34.461540
X33	170.000000	0.000000
X40	558.000000	0.000000
X41A	1040.000000	0.000000
X41B	0.000000	0.000000
X50	405.000000	0.000000
X51A	0.000000	24.038462
X51B	0.000000	24.038462
X60	0.000000	1.461538
X61A	1761.000000	0.000000
X61B	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
HIVOLHD)	0.000000	204.000000
MDVOLHD)	0.000000	148.000000
LOVOLHD)	0.000000	112.000000
CONIFHI)	0.000000	346.461548
MIXEDHI)	0.000000	264.000000
BARELND)	0.000000	62.461540
CONIFER)	0.000000	24.538462
TREATMNT)	413.000000	0.000000
FELLEDHD)	0.000000	0.029231
FELLEDOT)	0.000000	0.029000
12)	357.000000	0.000000
13)	197.000000	0.000000
14)	39.000000	0.000000
15)	0.000000	29.538462
16)	0.000000	28.923077
17)	0.000000	29.538462
18)	0.000000	-47.461540

NO. ITERATIONS= 5

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X10	204.000000	1.599997	31.333332
X11A	287.000000	19.199999	0.000000
X11B	215.000000	0.000000	INFINITY
X12	228.000000	34.461540	INFINITY
X13	292.000000	INFINITY	29.538462
X20	148.000000	28.923077	0.615383
X21A	201.000000	6.615386	INFINITY
X21B	135.000000	0.615383	INFINITY
X22	148.000000	35.076923	INFINITY
X23	212.000000	INFINITY	28.923077
X30	112.000000	0.999998	0.950001
X31A	157.000000	0.000000	10.750001
X31B	85.000000	47.461540	0.000000
X32	98.000000	34.461540	INFINITY
X33	162.000000	INFINITY	29.538462
X40	371.000000	38.461533	INFINITY
X41A	487.000000	INFINITY	0.000004
X41B	415.000000	0.000004	INFINITY
X50	264.000000	INFINITY	24.038458
X51A	337.000000	24.038460	INFINITY
X51B	265.000000	24.038458	INFINITY
X60	61.000000	1.461540	INFINITY
X61A	87.000000	INFINITY	0.000000
X61B	15.000000	0.000000	INFINITY

ROW	RIGHTHAND SIDE RANGES		
	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
HIVOLHD	2754.000000	INFINITY	1888.769287
MDVOLHD	850.000000	INFINITY	720.000000
LOVOLHD	850.000000	INFINITY	59.230770

CONIFHI	1598.000000	78.500000	38.500000
MIXEDHI	405.000000	INFINITY	405.000000
BARELND	1761.000000	78.500000	38.500000
CONIFER	3845.000000	38.500000	78.500000
TREATMNT	5000.000000	INFINITY	413.000000
FELLEDHD	2440000.000000	157000.000000	77000.000000
FELLEDDOT	4160000.000000	1651999.875000	4159999.750000
12	357.000000	INFINITY	357.000000
13	197.000000	INFINITY	197.000000
14	39.000000	INFINITY	39.000000
15	500.000000	38.500000	78.500000
16	130.000000	64.166664	130.000000
17	170.000000	38.500000	170.000000
18	500.000000	38.500000	224.285706