Let us consider the vector space $\mathbf{R}^m$ for convenience. Imagine you are given $k$ linearly independent vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ in $\mathbf{R}^n$ . We would like to find $m-k$ vectors $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{m-k}\}$ so that

$$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k, \mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{m-k}\} \text{ is a basis for } \mathbf{R}^n$$

There many ways to approach this. One way is to use Gaussiane eleimination techniques. Form an $m \times (m+k)$ matrix $A = [\mathbf{v}_1\, \mathbf{v}_2\, \ldots\, \mathbf{v}_k\, \mathbf{e}_1\, \mathbf{e}_2\, \ldots\, \mathbf{e}_m]$ where $\mathbf{e}_1\, \mathbf{e}_2\, \ldots\, \mathbf{e}_m$ is the standard basis for $\mathbf{R}^m$. Then $colsp(A) = \mathbf{R}^m$ and so a basis of the column space as reported by Gaussian elimination will be a basis of $\mathbf{R}^m$. Now you can check that Gaussian elimination must have the first $k$ columns as pivots (else there would be a dependency among $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$) and then we have a basis of $\mathbf{R}^m$ that contains $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$.

An alternate solution is to form a matrix $B = [\mathbf{v}_1\, \mathbf{v}_2\, \ldots\, \mathbf{v}_k]$ and apply Gaussian elimination (by multiplying $B$ by an invertible $E$) which yields a matrx $EB$ which has $m-k$ rows of 0's. Now append to $EB$ the $m-k$ columns $\mathbf{e}_{k+1}, \mathbf{e}_{k+2}, \ldots, \mathbf{e}_m$ so that the resulting $m \times m$ matrix $C$ has rank $m$. Now form $E^{-1}C$ which will also have rank $m$ and the columns of $E^{-1}C$ will be a basis for $\mathbf{R}^m$ and will be a basis including $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$.

If we are given an arbitrary $m$-dimensional vector space $V$ over field $\mathbf{R}$, we can choose a basis for $V$ and then coordinatize vectors so that we can manipulate them as vectors in $\mathbf{R}^m$.