Math 184 Problem

(from Charles Lamb)

A monopoly manufacturer estimates that when the price of an item it produces is \$100 then the weekly demand for the items is 3,000. For every \$1 increase in price, the weekly demand decreases by 30 items. Assume that the fixed costs of production on a weekly basis are \$72,000 and the variable costs are \$60 per item.

(a) Find the linear 'demand equation' for the item. Use the notation p for the unit price and q for the weekly demand.

(b) Find the weekly linear cost function $C = C(q)$.

(c) Find the weekly quadratic revenue function $R = R(q)$.

(d) Find the 'break-even' points where $C = R$.

(e) Graph $C = C(q)$ and $R = R(q)$ on the same set of axes with an eye to explaining why there are two 'break-even' points.

(f) Find the weekly quadratic profit function $P = P(q)$.

(g) Show that $P(q)$ on the graph in part (e) and indicate the regions of profit and loss (negative profit) on the q-axis.

(h) How should the monopoly company operate in order to maximize the weekly profit function $P = P(q)$? Give two ways of finding the correct answer.

(a) Find the linear 'demand equation' for the item. Use the notation p for the unit price and q for the weekly demand.

We are told that $p = 100$ and $q = 3000$ is on the line. We may think of price p as a function of q and obtain that the slope is $\frac{\Delta p}{\Delta q} = \frac{1}{-30} = 1\frac{1}{30}$. Thus using the standard point/slope formula (a line through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$) to get

$$p - 100 = -\frac{1}{30}(q - 3000) \text{ or } p = -\frac{1}{30}q + 200$$

This is the demand equation which we could also write (by rearranging) as

$$q = -30p + 6000 \text{ or } 30p + q = 6000$$

(b) Find the weekly linear cost function $C = C(q)$. The expression $C(q)$ is suggesting we want to write C as a function of q only.

$$C(q) = 72000 + 60q$$

(c) Find the weekly quadratic revenue function $R = R(q)$.

$$R = pq = (-\frac{1}{30}q + 200)q = -\frac{1}{30}q^2 + 200q$$

We could also express R as a function of p if we preferred.

(d) Find the 'break-even' points where $C = R$. We seek those q for which $C(q) = R(q)$ and so $72000 + 60q = -\frac{1}{30}q^2 + 200q$. This gives us a quadratic to solve:

$$\frac{1}{30}q^2 - 140q + 72000 = 0$$

which has solutions $q = 600$ or $q = 3600$. Both are legitimate solutions

(e) Graph $C = C(q)$ and $R = R(q)$ on the same set of axes with an eye to explaining why there are two 'break-even' points. The main idea here is that R yields a downward opening parabola while the line representing C cuts the parabola in two places.

(f) Find the weekly quadratic profit function $P = P(q)$. We use our vast accounting knowledge:

$$P(q) = R(q) - C(q) = -\frac{1}{30}q^2 + 140q - 72000$$

(g) Show that $P(q)$ on the graph in part (e) and indicate the regions of profit and loss (negative profit) on the q-axis. Given that we have a downward opening quadratic, the region of profitability will be those q between the two break-even points

(h) How should the monopoly company operate in order to maximize the weekly profit function $P = P(q)$? Give two ways of finding the correct answer. Determining the q which maximizes $P(q)$ is a standard one variable optimization problem. we could easily use Calculus (find q for which $P'(q) = 0$) but given that it is aquadratic we can readily use completing the square

$$P(q) = -\frac{1}{30}(q - 2100)^2 + 75000$$

from which we deduce that $P(q) \leq 75000$ and we get $P(q) = 75000$ only if $q = 2100$.