

2 Problem Set 2 — Graphical Analysis

- Use graphical analysis to describe all orbits of the functions below. Also draw their phase portraits.
 - $F(x) = 2x$
 - $F(x) = 1 - 2x$
 - $F(x) = x^2$
 - $F(x) = x - x^2$
 - $F(x) = \sin(x)$
 - Use graphical analysis to find all the points whose orbits tend to infinity, *i.e.* $\{x_0 \mid F^n(x_0) \rightarrow \pm\infty\}$, for the following functions:
 - $F(x) = x^2 + 1$
 - $F(x) = \begin{cases} 2x & 0 \leq x \leq 1/2 \\ 2 - 2x & 1/2 < x \leq 1 \end{cases}$
 - Completely analyse the orbits of the following functions:
 - $F(x) = \frac{1}{2}x - 2$
 - $F(x) = |x|$
 - $F(x) = -x^5$
 - $F(x) = e^x$
 - Analyse the orbits of the function $F(x) = |x - 2|$. Draw different types of orbits in different colours. You will be able to find fixed points, eventually fixed points, periodic points and eventually periodic points.
 - Let $F(x) = x^2 - \frac{6}{5}$. Find the fixed point(s) of F . Using the fixed point(s) (or otherwise) find the cycle of prime period 2.
 - Let $F(x) = ax + b$. Answer the following questions about the dynamics of F for various values of a and b :
 - Find the fixed points of F .
 - For what values of a and b does F have no fixed points?
 - For what values of a and b does F have infinitely many fixed points?
 - For which values of a and b does F have *exactly* one fixed point?
 - If F has exactly one fixed point and $|a| < 1$, what is the behaviour of all orbits under F ? Use graphical analysis.
 - Similarly, if $|a| > 1$ what is the behaviour of all orbits under F ?
 - If $a = 1$ describe the orbits of F for $b < 0$, $b = 0$ and $b > 0$?
 - Similarly, if $a = -1$ describe the orbits of F for $b < 0$, $b = 0$ and $b > 0$?
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