Blitz, Page 1

1. Evaluate the sum $(-5) + (-3) + (-1) + 1 + 3 + 5 + 7 + 9$.

1. _____

2. There are 7 marbles in a box, of which exactly 2 are white. One marble is selected at random. What is the probability it is non-white? Express the answer as a common fraction.

2. _____

3. The ferry left Tsawwassen at 2:33 PM, and arrived at Swartz Bay at 4:14 PM the same day. How many minutes was the trip?

3. _____ minutes

4. One angle of an isosceles triangle is $102°$. What is the measure, in degrees, of another angle of the triangle?
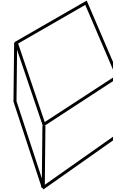
4. _____ degrees
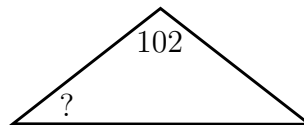
5. 10% of 20 plus 20% of 30 is how many % of 40?

5. _____ %

6. Express $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ as a common fraction.

6. _____

7. Round $\dfrac{230}{17}$ to the nearest integer.

7. _____

8. The sum of *all* the edge lengths of a cube is 36. What is the volume of the cube?

8. _____ units^3

9. What is the value of the smallest integer whose square root is greater than 5.5?

9. _____

10. Three fair coins are tossed. What is the probability of getting exactly 2 heads? Express the answer as a common fraction.

10. _____

11. What is the value of the sum

$$1000 - 999 + 998 - 997 + \cdots + 4 - 3 + 2 - 1?$$

11. _____

12. Two fair dice are rolled. What is the probability that the sum of the numbers obtained is 6? Express the answer as a common fraction.

12. _____

13. The combined cost of one candy, one chocolate bar, and one cookie is 2.71. The combined cost of one candy, one chocolate bar, and three cookies is 5.25. The combined cost of one candy, two chocolate bars, and three cookies is 6.36. What is the cost of one candy? Give the answer in \$, correct to two decimal places.

13. _____ \$

14. The 8 points below represent the vertices of a regular octagon. These vertices are alternately painted red and blue. Lines are drawn joining every pair of points. How many of these lines contain a red point and a blue point?

14. _____ lines
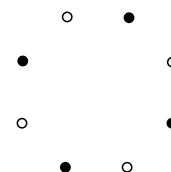
15. A number is put in each of the small rectangles below so that the number in any small rectangle is equal to the sum of the numbers in the two rectangles that it sits on. What number should be put in the rectangle labelled "?"?

15. _____

121
1	?	36
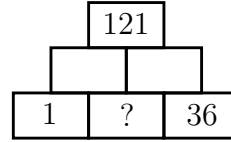
16. On Monday, 40 of the 50 students in the math class took a test. Their mean score was 60. On Tuesday, the remaining 10 students took the test. Their mean score was 95 (they had seen the questions). What was the mean class score on the test?

16. _____

17. Sprinter Bolt ran 100 metres in 10 seconds. What was his average speed in km/hour?

17. _____ km/hour

18. Last summer, Alfie earned $726 working at $11.00 per hour, and another $726 working at an hourly rate which is 50% higher. What was Alfie's average wage per hour last summer? Give your answer in dollars per hour, to 2 decimal places.

18. _____ $/hr

19. Four straight lines pass through a circular disk. What is the largest possible number of regions that these lines divide the disk into?

19. _____ regions

20. A lidless $4 \times 4 \times 4$ box is completely filled with $1 \times 1 \times 1$ cubes. How many of the $1 \times 1 \times 1$ cubes touch a side or the bottom of the box?

20. _____ cubes

21. When it was set out, the bowl of mixed nuts was (by weight) 45% peanuts, 25% almonds, 20% cashews, and 10% hazelnuts. Alicia picked out all the almonds and ate them. What percent (by weight) of the nuts in the bowl are now peanuts?

21. _____ percent

22. There is an integer N such that $N^3 = 79,507$. What is the value of N?

22. _____

23. Each line segment has length 1. You are only allowed to walk in the direction of the arrows, and must always walk full segments. At any intersection, you can walk in any allowed direction, regardless of where you came from. How many paths of total length 9 are there from A to B?

23. _____ paths
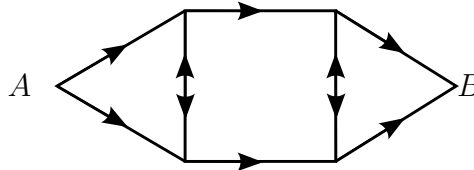
24. What is the x-coordinate of the point P on the x-axis such that the distance from P to the origin is the same as the distance from P to the point with coordinates $(1/2, 1/3)$? Express the answer as a common fraction.

24. _____

25. Two fair dice are rolled. What is the average value of the non-negative difference between the numbers showing on the two dice? Express the answer as a common fraction.

25. _____

26. In the diagram below, the two figures that look like squares are squares, with sides 12 and 13 respectively. What is the area of the shaded triangle? Express the answer as a common fraction.

26. _____ units^2
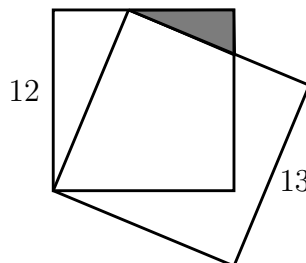
Bull's-eye, Page 1: Problem Solving

1. Assume that the Olympic village cost $1,120,000,000 and housed 2800 people during the Vancouver Olympics. What was the cost, in dollars, per person housed during the Olympics?

   1. _____ dollars

2. When Alicia's car is going at 90 km per hour, it uses 12 litres of gas to travel 100 km. How many litres of gas does it use per minute? Give the answer as a decimal, to 2 decimal places.

   2. _____ litres/min

3. It takes 12 hours to fill the pool with taps A and B (combined). It takes 36 hours with tap A alone, and it takes 8 hours with taps A, B, and C combined. How many hours would it take to fill the pool with tap C alone?

   3. _____ hours

4. A car is travelling at 96 km per hour. The radius of its wheels is $\dfrac{1}{\pi}$ metres. How fast are its wheels rotating in degrees per second?

   4. _____ deg/sec

Bull's-eye, Page 2: Numbers and Combinatorics

5. How many five-letter "words" are there which use only the letters A and/or B, and in which there are no consecutive occurrences of B? (For example, AAAAA and ABAAB qualify, but ABBAB does not.)

5. _____ words

6. Evaluate the cube root of $24 \times 30 \times 36 \times 40 \times 45$.

6. _____

7. To six decimal places, $\dfrac{\sqrt{5}-1}{2} = 0.618034$. What common fraction $\dfrac{a}{b}$, where $2 \leq b \leq 10$, is nearest to $\dfrac{\sqrt{5}-1}{2}$? For example, an answer of $\dfrac{4}{9}$ is wrong, but is of the right shape.

7. _____

8. Twenty-seven $1 \times 1 \times 1$ cubes are coloured using 27 different colours, one colour to each cube. Red and green are two of the colours. The 27 cubes are assembled at random to make a $3 \times 3 \times 3$ cube. What is the probability that the red cube and the green cube meet face to face? Express the answer as a common fraction.

8. _____

Bull's-eye, Page 3: Geometry

9. It is late afternoon, and a 1.2 metre tall child casts a 3 metre shadow. The child is standing next to an upright telephone pole, which casts a 40 metre shadow. What is the height, in metres, of the telephone pole?

9. _____ metres

10. In the chessboard below, the distance from point A to point B is 33 cm. What is the length, in cm, of a side of one of the 64 little squares on the chessboard? Express the answer as a common fraction.

10. _____ cm
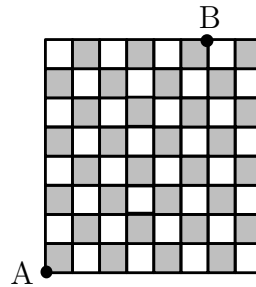
11. A line passes through the points $(-1, 10)$, and $(2, -9)$. If $5x + by = c$ is an equation of the line, what is the value of b? Express the answer as a common fraction.

11. _____

12. In $\triangle ABC$, the heights from A and from B are each equal to 4. Side AB is equal to 5. Given that $\triangle ABC$ is *not* right-angled, what is the perimeter of $\triangle ABC$? Express the answer as a common fraction.

12. _____ units
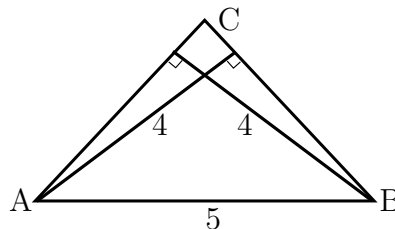
Co-op, Page 1: Team answers must be on the *coloured* page.
  Answers on a white page will not be graded.

1. The *digit-sum* of a positive integer is the sum of its decimal digits.   1. _____ integers
   For example, the digit-sum of 135 is 9. How many integers between
   100 and 999 have digit-sum equal to 5?

2. Define the sequence $a_1, a_2, a_3, \ldots$ as follows: $a_1 = 0$, and for every   2. _____
   positive integer n, $a_{n+1} = \dfrac{2}{1 + a_n}$. Thus for example $a_2 = \dfrac{2}{1 + 0} = 2$.
   Express a_5 as a common fraction.

3. How many perfect squares between 1 and 1,000,000 have 5, 6, 7, or   3. _____
   8 as their units digit?

4. Suppose that the line with equation $x + y = k$ passes through the   4. _____
   midpoint of the line segment that goes from the point $(1, 7)$ to the
   point $(9, 11)$. What is the value of k?

5. The sum of the ages of six brothers (all of different integer ages) is a   5. _____
   power of 5. The product of their ages is neither a multiple of 5 nor a
   multiple of 3. What is the least possible age of the oldest brother?

Co-op, Page 2: Team answers must be on the *coloured* page.
  Answers on a white page will not be graded.

6. For any real number x, $\lfloor x \rfloor$ (the integer part of x) is the greatest integer which is less than or equal to x. For example, $\lfloor 17.72 \rfloor = 17$, and $\lfloor 13 \rfloor = 13$. How many *different* integers are there in the list

$$\left\lfloor \frac{100}{1} \right\rfloor, \left\lfloor \frac{100}{2} \right\rfloor, \left\lfloor \frac{100}{3} \right\rfloor, \left\lfloor \frac{100}{4} \right\rfloor, \ldots\ldots, \left\lfloor \frac{100}{99} \right\rfloor, \left\lfloor \frac{100}{100} \right\rfloor ?$$

6. _____ integers

7. Scientists have recently discovered an Earth-like planet 128×10^{12} km away and decided to design a self-sustained spaceship that can travel for generations and send a human expedition to that planet. Based on current technology, spaceships can be designed to travel at a constant speed of 1.25% of the speed of light (after an initial stage of acceleration to this speed). Assume that the speed of light is 300000 km/sec, and ignore the time and distance the space ship has to travel until it reaches cruising speed. Assume also that a year has exactly 360 days and that the ship is to be launched in the year 2050 (exactly halfway through the 21st century). In what century will the space ship reach its destination?

7. _____ century

8. In Question #7, how many hours (rounded to the nearest whole hour) will it take for the spaceship to reach its cruising speed of 1.25% of the speed of light if it accelerates at a constant rate of 10m/sec^2.

8. _____ hours

9. In Question #8, how many km will the spaceship travel until it reaches its cruising speed? Give the answer in scientific notation, correct to 3 significant digits. So an answer like 6.24×10^5 is of the right shape.

9. _____ km

10. The number 360000 has 105 positive factors. (Note that 1 and 360000 are two of these factors.) How many of these 105 factors are divisible by 4?

10. _____ factors

Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

11. Ten people, Alan and Beti and 8 others, are divided at random into two groups, one with 4 people and the other with 6 people. What is the probability that Alan and Beti end up in the same group? Express the answer as a common fraction.

11. _____

12. There are 12 square tiles. Each has one letter written on it. The 12 letters are C, C, H, K, K, K, M, O, O, S, U, and U. The tiles are arranged to form the word SKOOKUMCHUCK. How many *distinct* "words" (which need not be words in any language) can be formed by interchanging two of the tiles? Note that SKOOKUMCHUCK itself is such a word, since it can be obtained by interchanging the two tiles that have a C on them, and in other ways.

12. _____ words

13. A regular hexagon is inscribed in a semicircle of radius 1 as shown. What is the area of the hexagon? Please give the answer rounded to 4 places after the decimal point.

13. _____ units^2

14. How many ordered pairs (a, b) are there such that a and b are positive integers and the least common multiple of a and b is 72? Remember that for example the ordered pair $(18, 24)$ is different from the ordered pair $(24, 18)$, and don't forget the ordered pair $(72, 72)$.

14. _____ pairs

15. A box contains N marbles, of which 2 are white and $N-2$ are black. You know that if you take out 3 marbles at random from the box, the probability that exactly 2 of them are white is $\frac{1}{210}$. What is the value of N?

15. _____