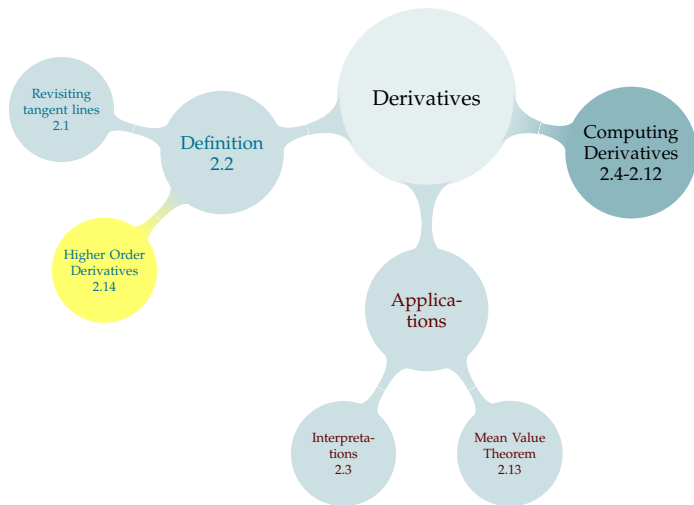


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# HIGHER ORDER DERIVATIVES

Evaluate  $\frac{d}{dx} \left[ \frac{d}{dx} [x^5 - 2x^2 + 3] \right]$

$$\frac{d}{dx} [x^5 - 2x^2 + 3] =$$

## Notation 2.14.1

The derivative of a derivative is called the **second derivative**, written

$$f''(x) \quad \text{or} \quad \frac{d^2 y}{dx^2}(x)$$

Similarly, the derivative of a second derivative is a third derivative, etc.

## Notation 2.14.1

- ▶  $f''(x)$  and  $f^{(2)}(x)$  and  $\frac{d^2f}{dx^2}(x)$  all mean  $\frac{d}{dx} \left( \frac{d}{dx} f(x) \right)$
- ▶  $f'''(x)$  and  $f^{(3)}(x)$  and  $\frac{d^3f}{dx^3}(x)$  all mean  $\frac{d}{dx} \left( \frac{d}{dx} \left( \frac{d}{dx} f(x) \right) \right)$
- ▶  $f^{(4)}(x)$  and  $\frac{d^4f}{dx^4}(x)$  both mean  $\frac{d}{dx} \left( \frac{d}{dx} \left( \frac{d}{dx} \left( \frac{d}{dx} f(x) \right) \right) \right)$
- ▶ and so on.

## TYPICAL EXAMPLE: ACCELERATION

- ▶ Velocity: rate of change of position
- ▶ Acceleration: rate of change of velocity.

The position of an object at time  $t$  is given by  $s(t) = t(5 - t)$ . *Time is measured in seconds, and position is measured in metres.*

1. Sketch the graph giving the position of the object.
2. What is the velocity of the object when  $t = 1$ ? Include units.
3. What is the acceleration of the object when  $t = 1$ ? Include units.

## CONCEPT CHECK

**True or False:** If  $f'(1) = 18$ , then  $f''(1) = 0$ ,  
since the  $\frac{d}{dx}\{18\} = 0$ .

**Which of the following is  
always true of a QUADRATIC  
polynomial  $f(x)$ ?**

- A.  $f(0) = 0$
- B.  $f'(0) = 0$
- C.  $f''(0) = 0$
- D.  $f'''(0) = 0$
- E.  $f^{(4)}(0) = 0$

**Which of the following is  
always true of a CUBIC  
polynomial  $f(x)$ ?**

- A.  $f(0) = 0$
- B.  $f'(0) = 0$
- C.  $f''(0) = 0$
- D.  $f'''(0) = 0$
- E.  $f^{(4)}(0) = 0$

# IMPLICIT DIFFERENTIATION

Suppose  $y(x)$  is a function such that

$$y(x) = y^3x + x^2 - 1$$

Find  $y''(x)$  at the point  $(-2, 1)$ .

## Included Work