

Math 615B - Term 2, Spring 2025

Introduction to Toric Varieties

Time and Location: MWF 11-12, MATH 202.

Instructor: Kalle Karu.
Office: 213 Math Bldg.
Phone: 822-4787.
E-mail: karu@math.ubc.ca.

Textbook: *Introduction to Toric Varieties* by William Fulton. UBC library has online access to the book.

Course description. The theory of toric varieties lies in the intersection of algebraic geometry and combinatorics. Toric varieties are certain algebraic varieties that can be defined combinatorially in terms of polytopes and fans. Because of their simplicity, difficult problems in algebraic geometry often reduce to much simpler combinatorial problems when restricting to toric varieties. Examples of these include intersection theory, Riemann-Roch theorems, resolution of singularities, minimal model program. On the combinatorics side toric varieties are very closely related to polytopes and more general polyhedral complexes. Many results in the combinatorics of polytopes have geometric interpretations in toric varieties and can be solved using algebraic geometry. For example: counting lattice points in a polytope, counting faces of a simplicial sphere, the scissor congruence of polytopes.

The course will cover most of the material in Fulton's book, plus some additional advanced topics .

Prerequisites. Some background in algebraic geometry will be assumed. Fulton's book is almost self-contained and one can read it as a first introduction to algebraic geometry. However, to appreciate the simplicity of toric varieties, it is good to know how difficult general algebraic geometry is.

Homework. There will be no exam. Homeworks will be posted approximately every second week.

Project. There will be a final project based on various research papers.

Marking. The course mark will be based on homework and the final project.