MATH 612: Topics in Mathematical Biology: biological image data and shape analysis

Khanh Dao Duc

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Summary

Advances in imaging techniques have enabled the access to 3D shapes present in a variety of biological structures: organs, cells, organelles, and proteins. Since biological shapes are related to physiological functions, biological studies are poised to leverage such data, asking a common statistical question: how can we build mathematical and statistical descriptions of biological morphologies and their variations? In this course, we will review recent attempts to use advanced mathematical concepts to formalize and study shape heterogeneity, covering a wide range of imaging methods and applications. The main mathematical focus will be on introducing the theoretical frameworks of Riemaniann geometry, diffeomorphisms and metrics over shape space, manifold learning, optimal transport theory with application for image analysis, with some other concepts covered in specific applications (e.g. quasiconformal mapping theory for shape representation, 3D reconstruction in Fourier space...). Specific biological imaging techniques, including atomic force microscopy, cryo-EM and cryo-ET, as well as computational methods and softwares will also be covered with guest lectures from expert practitioners. Students will be encouraged to work in groups to present research papers and do a small project to pass the course.

General information

Hours: Monday Wednesday Friday; 11am-12pm
Hybrid format: Zoom / ESB 4127 (PIMS)
Contact: DM on Canvas preferred (email: kdd@math.ubc.ca)
This course if offered as a PIMS network course: Students at Canadian PIMS member universities may apply for graduate credit via the Western Deans' Agreement (WDA).

Evaluation

1. (30%) Students will be required to write a blog post on Quarto[1] detailing some aspects of the course based on the lectures.

2. (70%) Students can opt to write a review or present a small research project on a topic that is relevant to the course.

3. Additional marks will be awarded for attendance/ Participation

Lecture schedule

Part 1 (week 1-3): Introduction to shape space

1. Sep 4 W: General Introduction
2. Sep 6 F: The Kendall Shape space
3. Sep 9 M: Elements of Riemaniann geometry
4. Sep 11 W: Manifold structure of the Kendall shape space
5. Sep 13 F: Statistical analysis on shape data (mean, PCA)
6. Sep 16 M: Introduction to Geomstats (Python package for shape analysis) (guest lecture by Adele Myers (UCSB)[2])
7. Sep 18 W: Discussion on Projects / Hands on session for Quarto / Github

Part 2 (week 4-6): Processing of shape data and biological applications

1. Sep 20 F: Guest lecture by Ben Cardoen (UBC, Biochemistry)
2. Sep 23 M: Guest lecture by Robert Ivan Nabi (UBC, Biochemistry[3])
3. Sep 25 W: Diffeomorphisms on curves and surfaces - The Riemannian elastic metric on cell shapes
4. Sep 27 F: Cell shape descriptors (guest lecture by Ashok Prasad Colorado State University[4])
5. Oct 2 W: Geodesic regression on brain shape (Guest lecture by Adele Myers (UCSB)[5])
6. Oct 4 F: Introduction to cell shape processing (segmentation, skeletonization)

[1] `https://bioshape-analysis.github.io/blog/`
[2] `https://ahma2017.wixsite.com/adelemyers`
[3] `https://cps.med.ubc.ca/faculty/nabi/`
[4] https://www.engr.colostate.edu/cbe/people/ashok-prasad/
[5] `https://ahma2017.wixsite.com/adelemyers`

7. Oct 7 M: Point clouds and Tessellation methods
8. Oct 9 W: Alpha shapes and applications (guest lecture by Wenjun Zhao (UBC[6]))
9. Oct 11 F: Application to Atomic Force Microscopy for microbiology (guest lecture by Haig Alexander Eskandarian Harvard Medical School)

Part 3 (week 7-11): Mathematical framework for structural biology

1. Oct 16 W: Introduction to cryo-EM and tomography
2. Oct 18 F: Fourier slice theorem and 3D reconstruction
3. Oct 28 M: Blog post presentation / project discussion
4. Oct 30 W: Atomic model building and AlphaFold
5. Nov 1 F: Heterogeneity analysis in cryo-EM (guest lecture by Geoffrey Woollard UBC)
6. Nov 4 M: Introduction to Optimal transport
7. Nov 6 W: Wasserstein metrics and computational Optimal Transport
8. Nov 8 F: Special lecture: Amit Singer (Princeton)
9. Nov 11 M: Applications to cryo-EM (Aryan)

Part 4 (week 11-13): Advanced representation of biological shapes with manifold learning and quasiconformal mapping

1. Nov 15 F: Introduction to manifold learning and Diffusion maps (1/2)
2. Nov 18 M: Introduction to manifold learning and Diffusion maps (2/2) Wenjun and KDD
3. Nov 20 W: Horizontal diffusion maps 1 (guest lecture by Wenjun Zhao and Shira Faigenbaum Golovin (Duke)[7])
4. Nov 22 F: Horizontal diffusion maps 2 (guest lecture by Wenjun Zhao and Shira Faigenbaum Golovin)
5. Nov 25 M: Horizontal diffusion maps: applications (guest lecture by Shira Faigenbaum Golovin)
6. Nov 27 W: Theory of quasiconformal maps
7. Nov 29 F: Un-Wrap Demo (guest lecture by Felix Zhou (UT Southwestern))
8. Dec 1 M: Surfaces in biological applications and datasets (Felix Zhou (UT Southwestern))

[6] `https://wenjunzhaowo.github.io`
[7] `https://services.math.duke.edu/~ag617/`