SYLLABUS FOR MATH 426, FALL 2023

1. References

The main reference for this course is the lecture notes which are posted on the course webpage. These notes are drawn from various sources including:

(1) *Topology and Geometry*, Glen Bredon
(2) *Algebraic Topology*, Allen Hatcher
(3) *Algebraic Topology: an Introduction*, W.S. Massey

2. List of topics and ideas

2.1. **Point Set Topology.**

(1) Definition of a topological space and continuous maps.
(2) Subspaces, interior, closure, dense sets.
(3) Induced topologies: subsets, quotients, products.
(4) Properties: Connected, path-connected, Hausdorff, compact. Locally path connected, locally simply connected, semi-locally simply connected.
(5) Homotopy equivalence of maps. Homotopy equivalence of maps relative to a subspace. Homotopy equivalence of spaces. Contractibility, deformation retracts.
(6) Basic notions of category theory: objects and morphisms, covariant and contravariant functors. Small categories. Groupoids. The homotopy category. Morphisms of groupoids.
(7) Mapping cylinders and mapping cones. The homotopy type of the mapping cylinder/cone only depends on the homotopy type of the map.

2.2. **Fundamental group and groupoid.**

(1) Homotopy classes of paths. Composition of paths.
(2) Definition of the fundamental groupoid $\Pi(X, A)$.
(3) Functorial properties of fundamental groupoid.
(4) Definition of the fundamental group $\pi_1(X, x_0)$. Dependence of the fundamental group on the base point.
(5) Fundamental groupoid of a contractible space.
(6) Van Kampen theorem for the fundamental groupoid.
(7) Van Kampen theorem for the fundamental group.
(8) Fundamental group of various spaces: S^1, S^n, $\mathbb{RP}^n$, $S^1 \vee \cdots \vee S^1$, surface of genus g, Klein bottle.
(9) Free groups, group presentations, amalgamated product of groups.
(10) Fundamental group of knot compliments, Wirtinger relations.

2.3. **Covering Spaces.**

(1) Definition of a covering space. Elementary neighborhoods. Degree of a covering space.
(2) The path lifting property of covering spaces.
(3) The lemma concerning the uniqueness of lifts.

(4) The monodromy action of $\pi_1(X, x_0)$ on $p^{-1}(x_0)$ where $p : \widetilde{X} \to X$ is a covering space.
(5) The category of covering spaces, in particular, isomorphisms of covering spaces and automorphism of a covering space (deck transformations). Galois/normal covering spaces and the relationship with quotients by the action of a group that acts properly discontinuously.
(6) The correspondence theorem between isomorphism classes of covering spaces and conjugacy classes of subgroups of the fundamental group.
(7) General solution to the lifting problem.
(8) Definition of the universal cover. Existence of the universal cover (for connected, locally path connected, and semi-locally simply connected spaces).
(9) The group of deck transformations of $p : \widetilde{X} \to X$ is isomorphic to $N(p_*\pi_1(\widetilde{X}, \tilde{x}_0))/p_*\pi_1(\widetilde{X}, \tilde{x}_0)$.
(10) You should know several examples of covering spaces. We saw examples involving graphs, surfaces, and quotients of spaces by discrete groups.

2.4. **Cell complexes, graphs.**

(1) Definition of CW complexes (a.k.a. cell complexes). Euler characteristic of a finite CW complex.
(2) Definition of a graph. Definition of directed edges, cycles, trees, valence.
(3) Trees are contractible. A finite connected graph Γ is homotopic to a bouquet of $1 - e(\Gamma)$ circles.
(4) Covering spaces of graphs are graphs. Corollary of covering space theory: an index d subgroup of a free group on N generators is isomorphic to a free group on $d(N-1)+1$ generators.
(5) Construction of a 2 dimensional CW complex whose fundamental group is any given finitely presented group.
(6) Definition of a $K(G,1)$ a.k.a BG (where G is a discrete group).