Course outline

MATH 400 **Applied Partial Differential Equations**, University of British Columbia
2023 Winter Term 2 (January - April 2024)

Description

In this course we concentrate on analytic methods to solve partial differential equations (PDEs) coming from physical applications. We will study first order equations, wellposedness and classification of second order PDEs, separation of variables and Sturm-Liouville theory, heat, wave and potential equations, PDEs in polar and spherical coordinates, integral transform methods, and examples of nonlinear second order PDEs.

Prerequisites

PDE 1: One of MATH 256, MATH 257, MATH 316, MATH 358, MECH 358, PHYS 312
Complex variables: One of MATH 300, MATH 305

Topics

1. First Order Equations: (1.2, 1.3) (5 lectures)
    - Simple transport; Linear first order equations; Characteristics and coordinates; Variable coefficients; Traffic flow; Dimensional analysis and scaling; Quasilinear equations.
2. Linear Second Order Equations: (1.6) (2 lectures)
    - Introduction to PDE. Initial and boundary conditions.
    - Well-posedness, Ill-posedness. Examples of Ill-posedness.
    - Classification of 2nd order PDE in 2D, change of variables.
3. The Wave and Diffusion (Heat) Equations: (3 lectures)
    - Vibrating string; Wave equation; (1.3, 2.1, 2.2, 3.2, 3.4)
    - Heat flow; Diffusion (heat) equation. (1.3, 2.3)
4. Separation of Variables and Sturm-Liouville Theory: (2 lectures)
    - Separation of variables; (4.1)
    - Sturm-Liouville boundary value problems; Sturm-Liouville theory and applications. (11.5, 5.3, 5.4)
5. Laplace’s (Potential) Equation: (2 lectures)
    - Temperature distributions; (1.3)
    - Laplace’s (potential) equation; Harmonic functions; Poisson’s formula. (6.1, 6.2, 6.3, 6.4)
6. PDE in higher dimensions: (7.1, 10.2, 10.3) (4 lectures)
    - Vibrations of a drum; Laplace’s equation in a solid ball; Spherical harmonics, special

functions.

7. Integral Transform Methods: (4 lectures)
    - Fourier transforms and applications; (12.3)
    - Laplace transforms and applications. (12.5)
8. Introduction to Nonlinear Equations: (3 lectures)
    - Modal truncation;
    - Reaction-diffusion equations.
9. total: 25 lectures

References

There is no required textbook for this course. I will refer to the following optional books for relevant materials:

- Walter A. Strauss, Partial Differential Equations, An Introduction, John Wiley & Sons, Inc., 2004 2nd ed. This has been the **main and only reference** of MATH 400 by several other professors. I know Prof. Strauss personally and have great respect for him.
- Peter J. Olver, Introduction to Partial Differential Equations, Springer 2014. I have not used this book before, but I read several good opinions about it in the internet. A great thing about this book is that **its ebook is available in the UBC library**. I attended several lectures of Prof. Olver while I was a graduate student in Minneapolis.
- Richard Haberman: Applied Partial Differential Equations, Fourth edition, Pearson Prentice Hall, 2004. It was the main reference of MATH 400 when I taught this course in 2004.

Important Dates

- First lecture: Tuesday, Jan. 9
- Last day to withdraw without record: Friday, Jan. 19
- midterm break: Feb 19-23
- Midterm exam: Thursday, March 7
- Last lecture: Thursday, Apr. 11
- Final exam: TBA

Grading

Homework 16%

Ten (10) weekly homework due Thursdays Jan 25, Feb 1, 8, 15, 29, Mar 14, 21, 28, Apr 4, 11 at 11:59pm on Canvas, with the lowest score dropped;

Midterm exam 28%

One 75-minute midterm exam on Thursday, March 7, in class.

Final exam 56%

One 150-minute final exam. The final exam is accumulative and covers all topics.

Canvas

Canvas is UBC's mobile-friendly online learning platform. You can log in Canvas with your CWL id. Announcements, assignments, lecture notes, solutions of homework and exam will be all posted in Canvas. You will take photo or scan your assignments and upload them to Canvas.

Piazza

We will have a forum at Piazza (see link on the sidebar, or visit **this Piazza link (https://piazza.com/ubc.ca/winterterm22023/math4002012023w2)**). You can ask and answer questions there. It is more efficient than emailing questions to the instructors since many students will have similar questions, the answers from your classmates may be easier to understand, and the process of discussion is also beneficial. Instructor and TA will occasionally check if there are questions unanswered. Following the advice of the math department, the set up is that students can show up as "Anonymous" to classmates, but not to instructors.

Policies on homework and midterms

1. No calculators or notes are allowed in the midterm and final exams.
2. Homework assignments are due 11:59pm at Canvas on Thursdays. Solutions will be posted on Canvas. A selection of the problems will be graded. If you submit homework late, a 25% penalty will be applied for each day late.
3. Permission to shift the weight of your missed midterm exam to the final exam, or to ignore missed assignments, may be granted only in the following circumstances:
   i. prior notice of a valid, documented absence on the scheduled date (e.g. out-of-town varsity athletic commitment with a letter from a coach), or
   ii. notification to the instructor of absence due to a medical condition with a doctor's note.

   Otherwise, a score of 0 will be given for the missed midterms/assignments. However, the **UBC policy on Academic Concession (http://www.calendar.ubc.ca/vancouver/index.cfm?tree=3,329,0,0)** allows students to request academic concession without documentations ONCE per course. For such request please fill the **form (https://owncloud.math.ubc.ca/index.php/s/mumsWsljdjR1idJ#pdfviewer)** .
4. The period for final exams is April 16-27, 2024 inclusive. The exact time will be announced by the University in the middle of the term. Students should not make early travel plans that overlap with the scheduled exam period.

Statement on UBC's Policies and Resources to Support Student Success

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including

those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious and cultural observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available **here (https://senate.ubc.ca/policies-resources-support-student-success)** .

Instructor

1. Tai-Peng Tsai, Math building room 109, phone 604-822-2591, email ttsai@math.ubc.ca , URL https://personal.math.ubc.ca/~ttsai/
2. office hours: Tue Thu 11-11:50am, Wed 2-2:50pm, and by appointments.