# Algebraic Geometry

**Time**:

Tuesdays and Thursdays, 14:00-15:20, **Auditorium Annex 142 (https://learningspaces.ubc.ca/classrooms/audx-142)**

## Contact:

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## Text:

We will roughly follow the first 11 chapters of the lecture notes by A. Gathmann, which are available at his website **here (https://agag-gathmann.math.rptu.de/de/alggeom.php)**

## Assessment:

There will be occasional homework assignments, some voluntary, some not. Your mark will be based on these.

## Prerequisites:

A one year graduate course in algebra is certainly highly desirable. A graduate course (or advanced undergraduate course) in differential geometry or topology would be advantageous, though not necessary.

## Goal:

This is a one-semester course in algebraic geometry. It is intended to be a first introduction to the field. The goal is to prepare students to take more advanced courses in the subject (such as 533 or a topics course), which could eventually lead to research in the algebraic geometry.

## Algebraic Geometry:

In a nutshell, algebraic geometry is the study of geometry by algebraic means.

For example, conic sections are described by their equations, such as $x^2 + y^2 = 1$. (This is a quadratic equation). Of course, we also consider higher order equations, such as cubics, $y^2 = x^3 + x + 1$. One of the most basic theorems in algebraic geometry is Bezout's theorem (18th century): the number of intersection points of two such algebraic curves in the plane, one of degree m and one of degree n is always nm. (for example, our above two curves should have 6 intersection points). There are many issues to be resolved, though, to make this always true. For example, some of the intersection points might have imaginary coordinates (this problem is resolved by working over the complex numbers). Or they might lie "at infinity". This is why we study "projective" geometry. (By the way, from the point of view of complex projective geometry all conic sections, ellipses, parabolas, hyperbolas look the same.) Finally, the curves might be tangent to each other, which means that such intersection points have ‘multiplicity’. (They may even coincide!)

As you can see, algebraic geometry is a very old subject, with a long history. But it is also a subject in which research is very active these days. Just recently (within the last 15 years), old questions such as ‘how many (parameterizable) curves of degree d in the plane pass through 3d-1 given points?’ have found answers, by the discovery of completely unexpected connections with quantum physics.

Algebraic geometry is also very important in number theory. The study of Diophantine equations (such as the solution of Fermat’s last theorem) is impossible without algebraic geometry.