MATH 516 Partial Differential Equations I

September - December, 2023

Course Description

This course is an introduction to the qualitative theory of partial differential equations (PDEs). It is offered every year, and should be useful to students with interests in applied mathematics, differential geometry, mathematical physics, probability, harmonic analysis, dynamical systems, and other areas, as well as to PDE-focused students. We will review a few analytic tools along the way, e.g., Fourier transform and weak convergence.

Prerequisites and corequisites

Undergraduate DE, Lebesgue integral and Lp spaces. (215/255, 257/316, 420/507).

Topics

Here is the tentative outline. It can be adjusted according to audience background and interests.

1. Classical linear equations
    a. Laplace, heat, and wave equations; their solution formulas
    b. mean value properties and maximum principles, applications to uniqueness and regularity
    c. existence: Perron's subsolution method and Dirichlet principle
    d. regularity of weakly harmonic functions, analyticity (sketch)
2. Classical solutions of second order elliptic equations
    a. weak and strong maximal principles
    b. Hölder spaces and Schauder's a priori estimates
    c. existence by the method of continuity
3. Sobolev spaces
    a. weak derivatives and Sobolev spaces
    b. inequalities of Sobolev, Morrey, Poincare, and Gagliardo-Nirenberg
    c. approximations, extensions, trace, compactness, and dual spaces
4. Weak solutions of elliptic equations in divergence form
    a. weak solutions and maximal principle
    b. existence and eigenvalues by Lax-Milgram theorem and Fredholm alternative
    c. regularity
    d. application to semilinear elliptic problems
    e. analogous results for 2nd-order parabolic equations
5. Semigroup theory and evolution equations
    a. semigroups
    b. applications to the existence of evolution equations (parabolic, wave, Schrödinger)

References

We will mainly follow Evans' book and also use materials from the others.

1. Partial Differential Equations, 2nd ed., by L. C. Evans, American Math Society, 2010. See author's homepage https://math.berkeley.edu/~evans for errata. This is a general text suitable for a first course and also for reference.
2. Partial Differential Equations, 4th ed., by Fritz John, Springer-Verlag. This is a classic textbook and contains materials not found elsewhere, e.g. Weyl's lemma and extended treatise on wave equation.
3. Elliptic Partial Differential Equations, by Qing Han and Fanghua Lin, Courant Lecture Notes 1, AMS/CIMS. This book is specialized in elliptic equations and is a standard reference. It is a good intermediate book after Evans and before Gilbarg-Trudinger.
4. Elliptic Partial Differential Equations of Second Order, 2nd ed., by David Gilbarg and Neil S. Trudinger, Springer-Verlag, Classics in Mathematics series. This book is specialized in elliptic equations and is a standard reference.

Evaluation

The evaluation is based on six (6) homework assignments and class participation.

Owncloud webfolder

Assignments and their solutions will be posted in an owncloud webfolder hosted by the math department. The link of the webfolder will be given to the audience. Also in the webfolder you will find my lecture notes and some references.

Instructor and lectures

Instructor: Dr. Tai-Peng Tsai, Math building room 109, phone 604-822-2591, ttsai at math.ubc.ca.

Lectures: MWF, 13:00 - 13:50, MATH 204

Office hours: TBA, and by appointment (Tsai's schedule).

Department of Mathematics | *University of British Columbia*

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