Real Analysis II/Functional Analysis

Course Code: Math 421/510 - Spring 2024

University of British Columbia

Acknowledgement

Our course is held on the UBC Point Grey (Vancouver) campus, which sits on the traditional, ancestral, unceded territory of the xʷməθkʷəy̓əm (Musqueam) First Nation.

Outline

This is a cross-listed 4th year undergraduate, 1st year graduate course. It is worth 3 credits.

The course is an introduction to Functional Analysis, which in a nutshell is the study of infinite-dimensional vector spaces with a norm giving rise to a topology, and the linear operators between them. The name "Functional Analysis" is a bit of a misnomer, since the course is not really about functions, but rather about vector spaces and linear operators. The name comes from the fact that the most classical/important examples of such spaces are spaces of functions.

Functional Analysis is then a generalization of linear algebra but, unlike the finite-dimensional case, the theory is much richer and more subtle, with topology playing a prominent role. It is also a very useful tool in many areas of mathematics, including PDEs, mathematical physics, probability, harmonic analysis, analytic number theory and combinatorics. It is also a beautiful subject in its own right, with many deep and surprising results.

Instructor and office hours

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Prerequisites

Math 420/507 (Real Analysis I) or equivalent background in measure theory. Familiarity with metric spaces and linear algebra is expected.

List of topics

The main topics to be covered, subject to small changes, are the following:

1. **Topology basics**: Topological spaces, continuity, compactness, Urysohn's lemma.
2. **Banach spaces**: Normed spaces, completeness, spaces of continuous functions (Arzela-Ascoli and Stone-Weierstrass theorems), L^p spaces, bounded operators.
3. **Hilbert spaces**: Inner product spaces, orthonormal bases.
4. **Topology of bounded operators**: Uniform boundedness principle, open mapping and closed graph theorem, Baire's theorem.
5. **Compact self-adjoint operators**: Definition and examples, the adjoint operator, the spectral theorem.
6. **Dual spaces**: The Hahn-Banach theorem and its applications. The dual of L^p. Weak topologies and the Banach-Alaoglu theorem.
7. **Additional topics**: if time allows, the Riesz representation theorem and/or the Krein-Milman theorem.

Course practice and assessment

There will be weekly homework assignments, available on Canvas each Friday starting on January 12, and due the following Friday at 9pm via Canvas upload. Only one (clearly identified) problem per assignment, possibly with multiple parts, will be graded. You are welcome to submit the solutions of up to two other problems of your choice; these will not count for marks, but you will get feedback from the TA. Solutions to all problems will be provided.

The course grade will be determined as follows:

- **Homework**: 20% (the single problem that is graded per week). Both the mathematical correctness and the clarity of the exposition will be taken into account. Late HW will not be accepted, but the lowest HW score will be dropped to account for short illnesses, etc. You are allowed and encouraged to discuss problems with your classmates, but you must write your own solution to the problem to be marked. Homework must be typeset in LaTeX, and submitted as a PDF.

- **Midterms**: 20%. There will be two midterms, on February 12 and March 14, in class, each worth 10% of the total grade. The problems in the midterms will be taken from among the non-graded problems in the previous homework assignments. If you must miss a midterm, the weight will be transferred to the final exam. There are no make-up midterms.

- **Final exam**: 60%. One sixth of the final exam (10% of the final grade) will consist of homework problems from after the second midterm. The remaining 5/6 of the final exam (50% of the total grade) will be cumulative. The final exam will be held during the final exam period, at a date and time to be announced by the university. The rules concerning the final exam, including missed exams, are those set by the university.

Bibliography

We will follow parts of [M. Einsiedler and T. Ward, *Functional Analysis, Spectral Theory, and Applications*]. This book is available online through the UBC library.

Other useful resources are:

- Notes from Math 421/510 by Prof. Sven Bachman. Available on Canvas with Sven's permission. The topics we will cover are very similar, but the order and emphasis will be different.

- The course topics also have a lot of overlap with Chapters 4,5,6 from the book [G.B. Folland, *Real Analysis: Modern Techniques and Their Applications*].

- There are many other excellent textbooks; see the bibliography in Einsiedler-Ward for some suggestions.

Bad weather policy

If UBC is subject to extreme winter weather conditions, the administration may decide to curtail in-person learning activities and other non-essential services. In such an event, **classes and office hours for Math 421/510 for that day will be canceled**.

Even in the absence of an official announcement, I reserve the right to use my own judgemment and move the lecture online at short notice if I feel that the weather conditions make it too dangerous or too difficult to travel to campus. In such a case, I will send an email to the class and post an announcement on Canvas. In this eventuality, the lecture will be recorded and posted on Canvas.