Welcome to Math 300, the UBC introductory course on one complex variable.

Tue-Thu 14:00-15:30 in **IRC-4 (https://learningspaces.ubc.ca/classrooms/irc-4)**

Please wear a mask to class if you have a cough or any cold/flu/covid-like symptoms.

Contact information:

- Instructor: Kai Behrend, Math 227, **behrend@math.ubc.ca (mailto:behrend@math.ubc.ca)** (no Canvas emails, please)

  Office hours: Tue 15:30-16:30, Wed 14:00-15:00

- TA: Yucheng Liu **yliu135@math.ubc.ca (mailto:yliu135@math.ubc.ca)**

  Office hours: Mon 15:00-16:00, Thu 15:30-16:30 in LSK 300 B

- TA: Souptik Mudi **smudi123@math.ubc.ca (mailto:smudi123@math.ubc.ca)**

  Office hours: Mon 12:00-13:00, Thu 12:30-13:30 in LSK 300 B

Learning Materials:

We will use the following three sources:

- John M. Howie: **Complex Analysis.** Springer Undergraduate Mathematics Series. Available online at the **UBC library. (https://go.exlibris.link/Gyv59V6D)**

This will serve as our main text, which we follows most closely. We will skip some of the more technical parts, and most proofs. But we will also add some material.

- M. Beck, G. Marchesi, D. Pixton, L Sabalka: **Complex Analysis.** Free open textbook available at **GitHub. (https://matthbeck.github.io/complex.html)**

This is an alternative to Howie, which you may prefer. It has a lot of homework and practice problems.

- Elias Wegert: **Visual Complex Functions.** Birkhäuser. Available online at the **UBC library (https://go.exlibris.link/q3PSQDr8)** .

This book is mostly included for its visualizations. Otherwise it is a bit too advanced for this course.

Lecture notes will be published here on Canvas.

Practice midterms and finals will be posted closer to the respective exams.

Temporary link to **lecture notes. (https://notability.com/n/2VH47xx6l3A84XDS4dggCp)**

Piazza:

we will use Piazza for discussion of mathematical content. Ask (and answer!) your questions about homework and midterms on Piazza.

Visualizations:

See **n-th roots (https://complex-analysis.com/content/roots_complex_numbers.html)** of complex numbers in action.

See **real and imaginary parts (https://complex-analysis.com/content/complex_functions.html)** of complex functions in action.

Play around with **phase portraits (https://complex-analysis.com/content/domain_coloring.html)** or **analytic landscapes (https://complex-analysis.com/content/analytic_landscapes.html)** of complex functions.

Pointwise (https://www.geogebra.org/m/bQkM5CR2) complex function visualizer

Stereographic projection of **circles (https://www.geogebra.org/m/vb6vuwbk)** and **triangles (https://www.geogebra.org/m/PVSq4f7Q)**

Taylor seres (https://complex-analysis.com/content/taylor_series.html)

A few random **Riemann surfaces (https://personal.math.ubc.ca/~behrend/math300/branchcuts2023.html)**

Homework:

Homework will consist of a combination of webwork and written homework. There may be some written homework which will not be marked (depending on TA availability), but unless mentioned otherwise, assignments will be marked and count towards your grade.

Exams:

There will be two midterm exams (in class) and one final exam.

- **Midterm 1:** Tuesday, February 13
- **Midterm 2:** Thursday, March 21
- **Final Exam:** to be scheduled by the university

All exams are closed book. No outside sheets/papers/notes/books are allowed. Also, no electronic devices of any kind are allowed. This includes all types of calculators. A formula sheet will be provided with each exam.

Assessment:

Your course grade will be calculated by the formula

- Homework: 10%, MT1: 20%, MT2: 20%, Final: 50%

Note that grades may be scaled.

Course Policies:

No late homework will be accepted. There will be no make-up exams. Missing a midterm for a valid reason normally results in the weight of that midterm being re-distributed to the remaining midterm and the final exam. Any student who misses a midterm is to present the **Department of Mathematics self-declaration form (https://owncloud.math.ubc.ca/index.php/s/mumsWsljdjR1idJ#pdfviewer)** for reporting a missed assessment to their instructor within 72 hours of the midterm date. This policy conforms with the UBC Vancouver Senate's Academic Concession Policy V-135 and student are advised to read this policy carefully.

You are encouraged to discuss homework with your classmates, but do not present other peoples' work as your own.

General Information. (https://www.math.ubc.ca/general-syllabus-information)

Material Covered

Part I

(9 lectures. This material will be tested in Midterm1 and the final exam.)

Complex numbers, complex functions, differentiability, power series, logarithms

- Complex numbers. Howie: 2.1, 2.2, BMPS: 1.1, 1.2, 1.3, (Wegert: 2.1, 2.2, 2.3)
- Complex functions, limits, continuity, analytic landscapes, phase portraits Howie: 3.1, 3.2, 3.3, BMPS 2.1, 7.1, (Wegert 2.2, 2.4, 2.5)
- The extended complex plane, the Riemann sphere, stereographic projection, BMPS: 3.3
- Moebius transformations, the cross ratio. Howie 11.3, BMPS: 3.1, 3.2, (Wegert 2.3, 6.3)
- Differentiability, the Cauchy-Riemann equations, holomorphic functions, Howie: 4.1, BMPS: 2.2, 2.3, 2.4
- Power series, Howie 4.2, BMPS: 3.4, 7.2, 7.3, 7.4, 8.1 (Wegert 3.2)
- Logarithms, branch cuts, Howie: 4.3, 4.4, BMPS: 3.5, (Wegert: 7.3, 7.4)

Part II

(8 lectures. This material will be tested in Midterm 2 and the final exam.)

Integration.

- parametrized curves and their traces, simple, closed, smooth curves, contours, oriented paths, Howie 5.2, BMPS 1.4
- integrals of complex functions, Howie 5.3, BMPS 4.1
- complex contour integrals, Howie 5.3, BMPS 4.1
- fundamental theorem of calculus for complex contour integrals, Howie 5.3, BMPS 4.2
- Cauchy's theorem, analogy with Greens' theorem, Howie 6.1, 6.2, 6.3, BMPS 4.3
- Cauchy's integral formula, Howie 7.1, BMPS 4.4
- Cauchy formula for higher derivatives, BMPS 5.3
- Infinite differentiability of holomorphic functions
- Morera's theorem
- Liouville's theorem, Howie 7.2
- fundamental theorem of algebra

Part III

(5 lectures. This material will be tested in the final exam.)

The residue theorem.

- Taylor series, Howie 7.4

- Laurent series, Howie 8.1, BMPS 8.3
- Singularities, Howie 4.5, 8.2, BMPS 8.2, 9.1 (Wegert: 4.4)
- residues and the residue theorem, Howie 8.3, BMPS 9.2
- computation of certain real improper integrals, Howie 9.1
- computation of certain series, Howie 9.5, BMPS 10.1
- argument principle, Howie 10.1
- Rouché's theorem
- winding numbers, Howie 10.3
- mean value property, BMPS 4.4
- maximum modulus principle, Howie 10.2, BMPS 8.2
- identity principle

Additional Topics

(2 lectures.)

- the Gamma function
- the Zeta function

Skills

This is a list of basic skills you are expected to master in this course.

A few items may be added as the course progresses.

(A few questions on exams may go beyond the basic skills.)

Complex numbers

- solve quadratic equations in $\mathbb{C}$
- solve radical equations $z^n = c$, where $c \in \mathbb{C}$,
- write complex expressions in the form $a + ib$,
- write complex expressions in the form $re^{i\theta}$,
- manipulate complex equations for lines and circles, using the method of "completing the absolute value squared"

Limits

- compute limits of functions at a point: $\lim\limits_{z \to z_0} f(z)$, including $z_0 = \infty$ or $\lim\limits_{z \to z_0} f(z) = \infty$,
- find limits of sequences of complex numbers: $\lim\limits_{n \to \infty} a_n$
- extend Möbius transformations to the extended complex plane $\mathbb{C} \cup \{\infty\}$,
- extend rational functions to the extended complex plane,

Möbius transformations

- find the inverse of a given Möbius transformation,
- find the formula for a Möbius transformation given where 3 points go,
- find Möbius transformations which map certain lines/circles/regions to other given lines/circles regions,
- find fixed points of Möbius transformations,
- convert simple geometric operations on the Riemann sphere into Möbius transformations,

Complex differentiation

- find the complex derivative of elementary functions,
- use the Cauchy-Riemann equations to find the locus of complex differentiability of a complex function,
- use the Cauchy-Riemann equations to find the region of holomorphicity of a complex function,
- use the Cauchy-Riemann equations to find the imaginary part of a holomorphic function given the real part (and vice versa),

Power series

- use the ratio, root and comparison tests to determine the (absolute) convergence behaviour of complex series $\sum_n a_n$,
- use the ratio and root tests to find the radius of convergence of a complex power series $\sum_n c_n(z-a)^n$,
- find the derivative of a holomorphic function defined by a power series,

Logarithms and powers

- find branches other than the principal branch of the complex logarithm,
- express (principal) branches of complex power functions $f(z) = z^c$ in terms of the principal branch of the logarithm,
- find the derivatives of branches of the logarithm and of power functions,

Contour integration of complex functions

For general functions $f(z)$:

- evaluate contour integrals $\int_\gamma f(z)\,dz$ by parametrizing the path γ,
- take advantage of reparametrizations when doing the previous,

For functions $f(z)$ admitting an antiderivative in their domain:

- evaluate contour integrals $\int_\gamma f(z)\,dz$ using the fundamental theorem of calculus for contour integrals (if f admits an antiderivative),
- take advantage of path independence, when possible,

For holomorphic or meromorphic functions $f(z)$:

- evaluate contour integrals using Cauchy's theorem,
- take advantage of path independence,
- evaluate contour integrals using Cauchy's integral formula,
- using partial fractions to evaluate integrals using Cauchy's integral formula
- compute integrals using the residue theorem
- compute integrals using the general residue theorem involving winding numbers

Laurent series

- find the annulus of convergence of a Laurent series from the coefficients
- find the Laurent expansion of a rational function in a given annulus by partial fractions and geometric series expansions
- find the orders of poles and zeros of a given meromorphic function
- find coefficients of Laurent expansions using integral formulas
- find coefficients of Laurent expansions by solving for them
- classify isolated singularities
- find residues by finding the Laurent expansion in a punctured disc
- find residues by formulas

Other

- compute certain real improper integrals using residue theory (Howie 9.1)
- use Rouché's theorem to find the number of zeros of a holomorphic function in a given region
- find winding numbers geometrically