Math 256 23W T2 Course syllabus

Course description

This course serves as an introduction to differential equations with a focus on solution techniques, transforms and modelling. Topics include first and second order linear ordinary differential equations, systems of first order linear differential equations, Laplace transforms, Fourier series and separation of variables for linear partial differential equations.

References (texts, course notes)

The course does not follow a particular textbook. An online textbook and a hardcopy textbook are listed on the Textbooks and course notes page here on Canvas (with links). The course is structured around lecture notes by Ian Frigaard, also linked on the same page. A rough breakdown is:

1st order linear DE's (~3 hours)

2nd order linear DE's (~7 hours)

Laplace transforms (~5 hours)

Systems of linear DE's (~6 hours)

Fourier series and simple BVP's (~3 hours)

Heat/diffusion, wave and Laplace's equations/separation of variables (~9 hours)

Marking scheme

WeBWorK assignments, approximately weekly - 10%

2 written long question assignments - 10%

2 Midterm exams - 15% each

Final exam - 50%

Homework

Homework for the course will come in two forms: (1) WeBWorK assignments, and (2) written long question assignments (to be scanned or created electronically and uploaded).

The WeBWorK questions will generally be short and are targeted simply so that you may practice the machinery of solving problems.

The written assignments will contain questions that may develop additional insights or explore applications in the format of multi-part questions. These are specifically designed to give you exposure to the type of questions that you may face on a midterm or on the final exam. The due dates for the first 2 written assignments are a few days before the 2 midterms. Solutions will be released and no late submissions accepted.

Written assignments are accessed and submitted using the GradeScope menu link.

Written assignment guidelines

For each written assignment, you will **upload a scan** of your written work to GradeScope.

Using a **tablet with writing app is fine** as long as you can export as a **well-formatted pdf thatappears as a letter sized page**. Points may be deducted for quirky difficult-to-navigate submissions.

Your work must be **legible and well-organized** enough for the markers to be able to read it and follow your logic without hesitation.

You must use a scanner or scanning mobile app to create a single small (<10MB) pdf file that has even lighting. Do not take jpg pictures and glue them together in Word. These tend to be difficult-to-read grey-scale images that are sometimes ridiculously large (100+MB).

(**video scanningdemo (https://canvas.ubc.ca/courses/131810/pages/scanning-demo)**)

Use letter-sized paper. Do not use graph paper unless the lines are not visible once scanned. You can use a tablet or similar to write up your answers provided your submitted pdf consists of separate letter-sized pages.

The TAs have been instructed to deduct 1 or 2 points for not adhering to these guidelines.

Missing midterms, exams, late homework

If you are unable to attend the midterm, you must notify your instructor before (preferred) or within two days after (in the case of emergencies) the exam date. In either of these two cases (and only in these two cases), suitable accommodations will be made. Generally, your final exam mark will be used in place of the missing midterm mark. Undocumented absence from the midterm will be given a score of zero.

The written homework will be due on the specified due dates at 11:59 pm. It is possible to submit up to one hour late without penalty but neither exceptions nor extensions beyond that will be granted.

No WebWork late submission will be accepted. The solutions will be released immediately after the due date.

DO NOT make any travel plans for April until the exam schedule is announced as no accommodation will be made for students unable to attend the final exam due to conflicting plans. Note that the exam period is densely packed and your exams could be scheduled any day of the week.

Prerequisites

First year calculus (MATH 100/101 or equivalent)

Linear algebra (MATH 152, MATH 221 or MATH 223)

Learning Goals

General knowledge of differential equations

i. Classify equations by linearity, order, ordinary/partial, non/homogeneous.
ii. Definitions: solution, general solution, particular solution.
iii. Verify that a given function is a solution to a given ODE.
iv. Determine arbitrary constants using ICs.

Module 1: First order equations

i. Solve constant coefficient inhomogeneous equations using method of undetermined coefficients (MUC)
ii. Determine correct integrating factor and solve equation.
iii. Plot integral curves of an ODE (whole family of functions parametrized by an arbitrary constant).
iv. Determine how many possibilities are there for different ICs, including parameter dependence.
v. Modeling: given a word description in a physically familiar area, write down ODE(s) and solve, e.g. Newton's Law of Cooling,

Module 2: Second order equations

i. Determine independence of functions using the Wronskian.
ii. Determine dependence of functions by finding a linear combination that sums to zero.
iii. Work with complex numbers and Euler's equation.
iv. Solve second order equations with constant coefficients: solve characteristic equation and translate into general solution: distinct real roots, repeated real root, complex roots.
v. Given form of ODE and the general solution, determine the details of the ODE.
vi. Use reduction of order to find a second solution when a first solution is given.

vii. Recognize the structure of solutions: where is homogeneous part + nonhomogeneous part reflected in solutions?
viii. Method of undetermined coefficients (MUC). ODEs with any RHS that has a finite family (or, bythe end of term, Fourier series) and cases in which RHS (or member of its family) solves the homogeneous equation.
ix. Determine correct form of for MUC (or work backwards to figure out unspecified RHS from given general solution).
x. Determine unknown coefficients in proposed y_p as per MUC.
xi. Be familiar with spring-mass systems

Module 3: Laplace transforms for solving second order equations

i. Laplace transforms (calculating them from the definition) and working with various general results to extend range of LT pairs
ii. Using partial fraction decomposition for Laplace inversion.
iii. Completing the square for Laplace inversion.
iv. Writing piecewise linear functions in terms of Heaviside-type functions ($u_c(t)$), piecewise functions involving ramps and other non-constant functions.
v. Solving IVPs involving various forcing functions (exp, trig, polynomial, piecewise) using the Laplace transform approach.
vi. Transfer functions, impulse response and convolution.
vii. Delta function properties - integrals involving delta functions, Laplace transform, relationship between delta (spikes), Heaviside (jumps) and absolute value (kinks/corners).
viii. Solving IVPs with delta function RHS.

Module 4: Systems of linear ODEs

i. Finding eigenvalues, eigenvectors and generalized eigenvectors.
ii. Writing down the general solution to systems with (i) real distinct eigenvalues, (ii) repeated real eigenvalues with a complete set of eigenvectors, (iii) repeated real eigenvalues without a complete set of eigenvectors (i.e. a defective matrix), (iv) complex eigenvalues.
iii. Find steady states (constant solutions).

iv. Relate vector field to system of equations, vector field to solution.
v. Solve inhomogeneous linear systems using MUC or variation of parameters
vi. Understand how Laplace transforms might be used for constant coefficient systems

Module G: Fourier series (FS)

i. Calculating FS.
ii. Using FS to solve second order ODEs with periodic forcing (Method of Undetermined Coefficients) and identifying resonance.
iii. FS for even and odd functions.
iv. FS convergence (e.g. for what values of x does the FS →f(x))
v. The conceptual difference between using FS for a function on (-,) and for a function on [0,L](e.g. how different extensions beyond [0,L] give different FS outside [0,L] but that agree on [0,L]).
vi. Using FS for simple boundary value problems

Module H: Partial Differential Equations

i. Simple derivation of the Heat/Diffusion equation.
ii. Using FS to solve the Heat/Diffusion equation. Homogeneous Dirichlet BCs Homogeneous Neumann (no flux).
iii. Simple derivation of the Wave equation.
iv. Using FS to solve the Wave equation. Homogeneous BCs & Non-homogeneous BCs and PDE: examples only.
v. Introduction to d'Alembert's method for wave equation
vi. Laplace's equation introduced
vii. Using FS to solve Laplace's equation