UBC Math 100

2023 WT1

Acknowledgement

UBC's Point Grey Campus is located on the traditional, ancestral, and unceded territory of the xʷməθkʷəy̓əm (Musqueam). The land it is situated on has always been a place of learning for the Musqueam people, who for millennia have passed on culture, history, and traditions from one generation to the next.

If you would like to know more about the joint history of UBC and Musqueam, one place to start is at UBC's Indigenous portal.

Course Information

Course Title	Course Code Number	Credit Value
Differential Calculus with Applications	MATH 100	3

MATH 100 involves both topics from differential calculus as well as multi-variable calculus. Students will learn the basic ideas, tools and techniques that they can use to solve problems with real-life applications.

Contacts

Do not email your instructor directly with questions. For questions about mathematics and homework, use the Piazza forum. The Math Learning Centre is also available for additional homework help.

For questions and issues regarding personal or administrative matters, use the Calculus Contact Form. This form can be found on your class Canvas site or here.

For office hours, please see Canvas → Sections.

Other Instructional Staff

We also have TAs who will moderate Piazza. Small section instructors will also hold regularly scheduled office hours.

Course Structure

There are three "flavours" of MATH 100, each with different applications.
Flavour "A" includes applications to Physical Sciences and Engineering.
Flavour "B" includes applications to Biology and Life Sciences.
Flavour "C" includes applications to Commerce and Social Sciences.
All flavours share the same homework and exams, and each large class will learn the same material. The instructors coordinate to teach roughly the same topics at roughly the same time. The differences are in the small classes, where each section may be taught examples of applications for their respective flavours.

Most weeks, you will attend one 2-hour lecture taught by the professor, and one 1-hour small class taught by a small class instructor and TA. There will be group work in small classes so attendance is mandatory, and will be taken.

Learning Objectives

Learning Objectives 1:

- Sketch basic power functions.
- Determine which term in a polynomial function will dominate for small x and for large x.
- Sketch two-term polynomial functions by determining which term dominates for small x and for large x. For example, sketch $f(x) = x^2 - x^4$.
- Sketch familiar functions such as e^x, $\log x$, $\sin x$, $\cos x$, $\tan x$, $1/x$, $\sqrt{x}$, and $|x|$.
- Demonstrate that e^x eventually dominates any given power function and use this fact for sketching. For example, sketch $f(x) = e^x - x^4$.
- Given a complicated function, describe in words the order in which to apply operations.
- Sketch rational functions of the form $\dfrac{Ax^n}{(B + x^n)}$ by considering behaviour for small x and for large x.
- Sketch products and ratios of familiar functions by separately considering component functions.

Learning Objectives 2:

- Explain using both words and pictures what $\lim_{x\to a} f(x) = L$, $\lim_{x\to a^-} f(x) = L$, and $\lim_{x\to a^+} f(x) = L$ mean (including the case where L is equal to ∞ or $-\infty$).
- Explain using using both words and pictures what $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to-\infty} f(x) = L$ mean (including the case where L is equal to ∞ of $-\infty$).
- Find the limit of a function at a point given the graph of the function.
- Evaluate limits of polynomial, rational, trigonometric, exponential, and logarithmic functions.
- Explain using both informal language and the language of limits what it means for a function to have a horizontal or vertical asymptote.
- Given a simple function, find its vertical and horizontal asymptotes by asymptotic reasoning or by taking limits.

- Use limits to find information about the shape of a function and in some cases produce a sketch.
- Explain why it is not true that a function cannot cross its horizontal asymptote.
- Explain informally what it means for a function to be continuous on its domain.
- Identify and classify points of discontinuity (jump, infinite, removable).
- Given a function defined with parameters, select parameter values that make the function continuous.
- Determine where a given function is continuous.

Learning Objectives 3

- Explain using words, pictures, and the language of limits what a derivative is.
- Use the definition of derivative to find the tangent line to a function at a given point.
- Describe the tangent line as an approximation to a function at a given point.
- Describe the derivative of a function as a function itself.
- Given the graph of a function, sketch the graph of its derivative.
- Describe the exponential function e^x in terms of its derivative.
- Explain what a differential equation is and provide solutions to the differential equation $y'(t) = y(t)$.
- Describe how the differential equation $y'(t) = y(t)$ can model population growth.

Learning Objectives 4:

- Demonstrate using the limit definition of derivative that differentiation is linear.
- Use linearity to "break down" derivatives of sums and constant multiples.
- Demonstrate the Power Rule for integer exponents using the limit definition of derivative.
- Use the Power Rule for integer exponents.
- Use counterexamples to demonstrate that certain statements about derivatives are false.
- Explain why an example does not constitute a "proof".
- Use the Product and Quotient Rules to differentiate the product or quotient of functions.
- Review the definitions and derivatives of trigonometric functions.
- Determine derivatives of trigonometric functions using the limit definition of derivative, trigonometric limits, addition formulas, and Product and Quotient Rules.

Learning Objectives 5:

- Use the Chain Rule to compute derivatives of compositions of functions.
- Differentiate logarithmic functions.
- Determine when to use logarithmic differentiation to simplify derivatives.
- Use logarithmic differentiation.
- Use the Generalized Product Rule to compute the derivative of products of many functions.

Learning Objectives 6:

- Explain how implicit differentiation is a consequence of the Chain Rule.
- Use implicit differentiation to find slopes of tangent lines to implicitly defined curves.
- Sketch and differentiate the inverse trigonometric functions $\arcsin(x)$, $\arccos(x)$ and $\arctan(x)$.
- Implement a sequence of steps to solve related rates problems.

Learning Objectives 7:

- Explain what it means for a twice-differentiable function to be concave up or concave down on an interval.
- Determine whether a twice-differentiable function is concave up or concave down on an interval.
- Explain how information about the graph of a function may be extracted from the function, its derivative and its second derivative.
- Sketch the graph of a function $f(x)$ using the function, its derivative and its second derivative.
- Sketch the graph of a function using characteristics determined from the function and its derivatives.

Learning Objectives 8:

- Recall what a linear approximation is.
- Explain what a degree n approximation of a function is.
- Determine degree n approximations for appropriately differentiable functions.
- State the Maclaurin polynomials for the standard functions: $\frac{1}{1-x}$, e^x, $\cos x$, $\sin x$, $\log(1+x)$.
- Note: we do not consider the error introduced by linear approximations or Taylor polynomial approximations.
- Derive the general formula for the nth order Maclaurin polynomial to $\frac{a}{1-x}$.
- Use the Maclaurin polynomial above to find Maclaurin polynomials for functions of the form $\frac{f(x)}{1-g(x)}$ where $f(x)$ and $g(x)$ are polynomials or other functions which we already have Maclaurin polynomials for (such as $\sin x$ or e^x).
- Find the limit of the partial sums S_n as $n \to \infty$ (we'll return to this in more detail in MATH 101).
- Use the above limit to make statements about certain infinite sums such as $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots = \lim_{n\to\infty}\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots+\left(\frac{1}{2}\right)^{n-1}\right)$.

Learning Objectives 9:

- Determine the critical and singular points of a function.
- Find the global extrema of a function on a closed interval.
- Explain how the algorithm can be used in optimization problems.
- Convert geometric information into a function optimization problem.
- Interpret model optimization problems based on real-world examples according to their context.

Learning Objectives 10:

- Explain how a differential equation is different from an algebraic equation.
- Check whether a given function satisfies a differential equation.
- Given an appropriate ansatz, find parameter values that satisfy a differential equation.
- Given an initial condition, find a particular solution that satisfies a differential equation.
- Identify solutions to simple differential equations (such as $y' = ay$ and $y' = a(y-b)$) and interpret them in context.
- Sketch slope fields (direction fields) for a given differential equation and use them to roughly sketch solutions.
- Sketch a phase line for a given differential equation and use it to describe the behaviour of solutions.

- Explain what is meant by a "steady-state" solution. Explain further what it means for a steady-state solution to be "stable".
- Find steady-state solutions to simple differential equations and determine their stability.

Learning Objectives 11 (Computational methods):

- Explain how a differential equation may be solved computationally using linear approximations. That is, explain how Euler's method works.
- Explain what each term represents in the formula for Euler's method.
- Examine and compare computational (numerical) and exact (analytical) solutions to differential equations.
- Use Euler's method to solve a differential equation both by hand (small number of steps) and by spreadsheet (large number of steps).
- Explain how Newton's method works. That is, how you can use tangent lines to approximate the roots of a function.
- Write down the formula for Newton's method and explain what each term in the equation represents.
- Use Newton's method to estimate the root(s) of a function.

Learning Objectives 12 :

- Label points on the x-y-z axes and identify basic planes of constant x, y, or z.
- Given a simple function of two variables, $z = f(x, y)$, evaluate z values for given pairs (x, y). State the domain and range of simple functions of two variables.
- Compute partial derivatives of two variable functions.
- Provide a physical interpretation of a partial derivative in terms of directional steepness (while at a point on a surface).
- Compute the second order partial derivatives given a function of two variables. State without proof that the mixed partials should be equal for "nice" functions.
- Plot points on the x-y-z axes.
- Given a function of two variables sketch level curves by setting z to be a constant.
- Given a relation between three variables, sketch traces by setting one variable to be a constant.

Learning Objectives 13

- Reduce a constrained optimization problem in 3D to several single variable calculus problems.
- Find the extreme values for a function of two variables on a closed region in cases where optimization on the boundary can be reduced to a single valiable caluclus problem.
- Define critical point and singular point for a function of two variables.
- Compute the critical points and singular points of a given function of two variables.
- Describe the process of finding the absolute maximum and minimum values of a two variable function on a closed domain.
- Given a function of two variables defined on a closed region, investigate the boundary in pursuit of absolute extrema. In particular, by eliminating one variable and using methods from single variable calculus.

Schedule of Topics

Week	Topics
September 11-15	Comparing power, log, exponential and trigonometric functions; parse trees; basic sketching
September 18-24	Rational functions; horizontal and vertical asymptotes; the language of limits; continuity
September 25-29	Definition and interpretation of the derivative; tangent lines; linear approximations; the exponential function and simple differential equations
October 2-6	The power, product and quotient rules; derivatives of trigonometric functions
October 10-13	The chain rule; the derivative of log(x) and logarithmic differentiation
October 16-20	Implicit differentiation and inverse trigonometric functions; related rates;
October 23-27	Curve sketching; the logistic model
October 30-November 3	Higher degree approximations; geometric series; differential equations: phase diagrams; ansatzes
November 6-10	Optimization
November 16, 17	Phase diagrams; ansatzes; revisiting the logistic model
November 20-24	Computation week: numerical derivatives; Euler's method, Newton's method
November 27-December 5	Introduction to multivariable functions, partial derivatives, second order partials; sketching in 3D; Local max and min, absolute max and min of multivariable functions

Learning Materials

Our course materials are linked to UBC's learning management system, Canvas.

This course uses the CLP-1 Differential Calculus textbook. You can find a book of practice problems in the second link on the same page. This is a free online textbook created by UBC professors for UBC students; there are no physical copies available but the PDF file is easily printable (given the link, copy shops will print and bind). At the end of the course, we will also refer to a few sections of the CLP-derived OIL textbook. We will occasionally refer to Differential Calculus for the Life Sciences.

Assessments of Learning

Grade Calculation

We believe that mastering technical skills is an essential component of this course. To encourage you to master these skills, we have implemented a benchmark system in which you must demonstrate mastery in order to unlock certain grades.

Your provisional course grade will be calculated as follows:

10% WeBWorK assignments
20% Tests
10% Written Assignments
10% Engagement
50% Final exam
Adjustment See Benchmark Requirements below.

Your final grade will then be the minimum of your initial final grade (as shown above) and your benchmark grades (as shown below).

All failing grades are double-checked before they are submitted. Final exams are not returned, but if you would like to view yours, you need to fill out the exam viewing form on the math department website.

WeBWorK: assignments and quizzes

There are 11 WeBWorK assignments. Only your top 10 WeBWorK assignments will be considered when calculating your provisional grade; this is intended to account for technical difficulties, illness, and other personal situations. WeBWorK assignments are due on Fridays. These assignments make up 10% of your initial grade. Only your highest attempt will be counted.

WeBWorK assignments are used to achieve technical and computational mastery. This is tested in Section 1 of the final exam. Section 1 consists of questions closely aligned to WeBWorK assignments.

WeBWork questions come in a number of different formats, and some questions will require you to type out an exact answer in order for the question to be correct. These answers are not always numbers, so making a typo could cost you marks. It is suggested that you click the PREVIEW ANSWERS button before you submit your work. If you have questions about formatting while you are doing your assignments or practicing, ask them on Piazza.

The WeBWorK quiz is not a weighted assessment, it will not contribute to your final grade. The quiz is a 50-minute timed assessment consisting of 25 WeBWorK questions selected from all previous WeBWorK assignments. It is open throughout the year. There is no limit to the number of times you may attempt the quiz. It is recommended that you attempt it at least once per week. You will get a new set of problems each time. The quiz closely matches Section 1 of the final exam. It constitutes an authentic "field test" of your readiness for that section of the final exam, and is meant entirely for practice.

In order to start the quiz, you must enter the access code found on the canvas course page. In order to submit the quiz, you must press the "Grade Test" button at the bottom of the quiz page. You must do this before the timer at the top of the quiz page reaches 0. Do not press this button before you wish to submit. Pressing the "Preview" button will save your work up to that moment. Do not open more than one WeBWorK tab while doing the quiz.

Written Assignments

Your participation in the small classes is mandatory.

There are four written group assignments. They will make up 10% of your initial grade. Group assignments are meant to explore and extend core concepts. Make sure to start early: it is not unusual to spend up to 12 hours on an assignment. Assignments are graded on clarity and coherence as well as correctness. They must be well written and clearly presented. Section 2 of the final exam includes questions based on written group assignments.

It is expected and encouraged for your group to work with other groups on these assignments. However, your group must write up its assignment independently, and any ideas inspired by discussions with students outside your group must be acknowledged in your submission.

Major group issues can be reported via the Calculus Contact Form, and should be done as early as possible in the semester, but note that students are largely expected to resolve these issues themselves.

Engagement

Engagement is worth 10% of your final grade.

Small classes: attendance and active participation within small classes comprises half of your engagement grade. Asking questions and contributing to team problem solving are good examples of active participation. You may be absent from one small class without penalty; this is to account for illness and other personal situations.

Reflection questions: The other half of your engagement grade is obtained from a reflective question asked at the end of each written assignment. This question is designed to help you reflect on the learned concepts and your progress within the course; the criteria to this question are not strict but your answer should be thoughtful and demonstrate your efforts to engage with the course material.

Other engagement grades: There may be other opportunities to participate throughout the term. In some cases, failing to complete a engagement task will result in a penalty. However, small class instructors may also give bonus engagement marks (to a maximum of 10 points) for especially productive engagement.

Benchmark Requirements

Exam Benchmarks

Your initial benchmark grade is 45%.

If you score 25/50 on Section 1 of the final exam, you unlock the benchmark grade of 100%.

This means if your benchmark grade is lower than your initial grade, your final mark in the class will be the benchmark grade.

For a full description of the final exam regulations, see the UBC Calendar page on Student Conduct during Examinations. Unless specifically stated otherwise, notes, calculators, cell phones and other electronic devices are strictly prohibited from use during the exam. This includes use of cell phones for checking the time.

Concessions

Concessions are handled slightly differently for different components of the course.

WeBWorK In previous years, when there were issues warranting a concession (technical problems, illness, family obligations, late registration, etc), students would contact their instructor to have the relevant assignment excused. In order to reduce this administrative burden, the score of your lowest WeBWorK assignment will not be considered in the calculation of your final grade. This is not intended as a grade-giveaway. It is intended to mitigate the paperwork involved in excusing assignments. It also avoids asking instructors and students to decide whose circumstances warrant exceptions. No late assessments are accepted.

Use the Calculus Contact Form if circumstances beyond your control, such as illness or extended family emergencies, cause you miss more than one WeBWorK assignment, or more than one quiz, you will also be directed to contact your faculty advising office to inform them of your circumstances.

Small Class Attendance Students are strongly encouraged to attend all small classes as you will have group members relying on you for your group work assignments. These classes are also where you will earn your engagement points, either through attendance or answering questions presented by your instructor. All students are excused from one small class attendance to account for illness and unexpected situations; you do not need to contact anyone in this situation. If you are absent from additional small classes please complete the calculus contact form.

Final exam The final exam is the most formal assessment, and subject to specific university regulations. Students unable to write the final exam must contact their faculty advising office.

Other Course Policies

General Syllabus Info

Information on general course policies including resources can be found here