Math 605D Tensor Decompositions and Applications

Fall 2022 - Syllabus

With the emergence of big data, it is more and more often the case that we encounter tensor-shaped data. The importance of being able to decompose a tensor is (at least) two-fold. First, finding the decomposition provides hidden information about the data at hand, and second, having a concise decomposition of the tensor allows us to store it much more efficiently. One of the biggest obstacles in dealing with tensors, however, is that decomposing them is often computationally hard.

This research-oriented course will introduce tensors (or multi-dimensional arrays) and their uses in statistics, machine learning, and the sciences. In particular, we will illustrate fundamental theoretical properties of several types of tensor decompositions, including CP-decomposition, nonnegative matrix and tensor decomposition, Tucker decomposition as well as tensor network decompositions arising from physics. We will see how these naturally come up in hidden variable models, Gaussian mixture models, directed and undirected graphical models, blind source separation, independent component analysis, and quantum physics. We will discuss algorithms for computing such decompositions, and will exhibit open problems.

Instructor: Elina Robeva, `erobeva@math.ubc.ca`, URL: `https://math.ubc.ca/~erobeva/`.

Class time: MWF 1-2pm Pacific Time.

Class location: Chemistry C124

Office hours: TBD

Class website: `https://sites.google.com/view/math-605d-tensors-2022/`

Prerequisites: Besides general mathematical maturity, the minimal suggested requirements for the course are linear algebra (e.g., one of Math 221, 223, 307), and basic probability (e.g., one of Math 302, 318). Some familiarity with machine learning is encouraged, but not required.

Bibliography: The main text we will use will be the lecture notes posted on the course website. We will also use a variety of book chapters and current papers. Some of these are listed at the end of this syllabus.

Lecture notes: Lecture notes and homework will be posted on the course website.

Grades: Research project 50%, Weekly reading report and discussion: 35%; Homework 40%.

Weekly reading reports and participation: There will be a research paper reading assignment every week. Together with a partner you are expected to submit a written report summarizing the paper, pointing out the main contributions, techniques, and adding your opinion on the quality of the work. We will discuss these papers during lecture every 2 weeks. Everyone will be expected to contribute to each discussion, and to lead one or two discussions during the term.

Research Project: This course includes a final project in which students address a topic of their choice. The goal is to explore a topic in-depth and to develop a related research problem. A one-page abstract describing the goals of the project, the main questions, and the approaches the students plan to take, is due midway through the term. Final presentations are during the last two or three lectures of the course and a final write-up of the project of maximally 10 pages is due at the end of the term. Students, preferably of different backgrounds, can pair up for the final project.

Homework: There will be a few light homework exercises assigned throughout the semester. You are encouraged to work together with others on the problems, but you have to write down the solutions on your own.

Collaboration policy: We encourage working together whenever possible. However, the handed in homework solutions should reflect each student's own understanding of the class material. It is not acceptable to copy a solution that somebody else has written.

References

The following references are relevant for the topics covered in this course.

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