MATH 546: Continuous Time Stochastic Processes
(3 credits)

Instructor: Mathav Murugan
Lectures: MWF 1-2
Classroom: Math Annex 1118
Office hours: TBA (will be indicated on Canvas)
Course webpage: We will use Canvas `https://canvas.ubc.ca/courses/112514`
Piazza: There is a link to Piazza on Canvas. Please use Piazza for questions that arise in your learning and for questions about all issues related to the course.
Text: *Diffusions, Markov Processes and Martingales 2nd Edition, Volume 2 Itô Calculus*, by L.C.G. Rogers and D. Williams, Cambridge University Press, 2000

Outline. This is a rigorous course on finite dimensional continuous Markov processes. Most topics covered will be included in Chapters IV and V of Rogers and Williams' text. We will study stochastic integration with respect to continuous semimartingales, and Itô's stochastic calculus. The focus of the course will be on finite-dimensional stochastic differential equations. After a brief review of Brownian motion we will study Itô's pathwise uniqueness results and then introduce the weak solutions, martingale problems and the relationship with strong or pathwise solutions. Change of measure (Girsanov) formulae will be derived and applied to the well-posedness of the martingale problem for finite dimensional sde's. Depending on the interest of the students we will then study local times and one-dimensional diffusion theory or Stroock-Varadhan martingale problems.

The course will assume familiarity with measure theoretic probability theory including discrete parameter martingale theory and Brownian motion although the course will start brief review of these last two topics. The text is self-contained for the most part but does refer to Volume 1 (cited below) on occasion. The plan is the following:

(1) Continuous time martingales and Brownian motion (3 weeks)
(2) Stochastic integration (5-6 weeks)
(3) Stochastic differential equations and other optional topics (4-5 weeks)

Prerequisites: Math 545 or consent of the instructor. Students from other Departments interested in learning about stochastic differential equations from a mathematical perspective are encouragedMeasure theoretic prerequisites may be treated as "black boxes".

Grading: This will be based on homework assignments which will be given every 2-4 weeks.

Other References:

[B] Breiman, Probability.
[D] Durrett, Probability: Theory and Examples.
[EK] Ethier and Kurtz, Markov Processes: Characterization and Convergence.
[L] J. F. Le Gall, Markov Processes: Characterization and Convergence.
[RY] Revuz and Yor, Continuous Martingales and Brownian Motion.
[P] Protter, Stochastic Integration and Differential Equations.
[W] D. Williams, Probability with Martingales.
[W1] D. Williams, Diffusions, Markov Processes and Martingales Vol. 1.

Updated: September 6, 2022