

Math 521: Finite Element Methods

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Course Description:

Over the last few decades, finite element methods have been used for the approximation of solutions to large classes of partial differential equations arising in fluid dynamics, solid mechanics, and electromagnetics. This course is an introduction to the mathematical theory of finite element methods. We will introduce finite element discretizations for equations of various types and discuss how these discrete problems can be solved efficiently. We will address mathematical questions related to the concepts of consistency, stability, convergence, and error estimation. Implementation will be done with freeware finite element packages. Preliminary insight into convergence analysis and the behaviour of iterative linear algebra solvers will be seen using Finite Difference and Spectral spatial discretizations.

The outline of the course is as follows:

- Warm up: Floating point representation, condition number, and root finding.
- Functional approximation: interpolation and quadrature. Introduction to functional analysis.
- ODE boundary value problems:
 - Finite Difference, Finite Volume, and Spectral Methods
 - Weak formulation
 - Finite Element Method (FEM) specification
 - FEM convergence analysis
- FEM for 2D and 3D elliptic problems:
 - Elements
 - Analysis
 - Implementing boundary conditions
- Time-dependent problems
- Numerical Linear Algebra: Conjugate Gradient and Multi-Grid Methods.
- Final topics, as time permits:

- Finite element methods for incompressible fluid flow
- Discontinuous Galerkin methods
- Error estimation and adaptive methods

Text:

There will be no prescribed text, but there will be lecture notes available for most parts of the course material. Some optional references will be listed.

Prerequisites:

Some undergraduate level training in at least one of: partial differential equations, analysis, or numerical analysis.

Assessment:

There will be five challenging homework assignments involving both analysis and computation. In addition, students will do a course project. Assignments are worth 60% of the final grade, the project 40%.