# MATH 515
# Partial Differential Equations of Fluid Mechanics

January - April, 2023
https://personal.math.ubc.ca/~ttsai/courses/515-23Q1/

## Course Description

This course introduces the mathematical theory for the partial differential equations (PDE) modeling the inviscid and viscous incompressible fluids, namely, the incompressible Euler equations and incompressible Navier-Stokes equations. A graduate student in either engineering, mathematics, or physics can hope to learn an important branch of the PDE theory motivated by fluid mechanics.

Because some registered students are from Applied Science, I plan to be sketchy on the proofs, but give more ideas and examples to the results we will present. The topics could be adjusted according to audience background and interests.

## Prerequisites

The required mathematical background will be covered in the first part of the course. Most of them are covered in MATH 516, which is encouraged. However we will not assume MATH 516, in order to broaden the audience base.

## Topics

Here is the tentative outline. It can be adjusted according to audience background and feedback.

1. Mathematical background
    1. Lebesgue integral and L^p spaces
    2. weak derivative and Sobolev spaces
    3. weak convergence
    4. solutions for the heat equation in a domain, Galerkin and semigroup methods

2. An introduction to incompressible fluid flows
    1. derivation of the Euler and Navier-Stokes equations
    2. symmetry groups and conserved quantities; some exact solutions
    3. Leray's formulation and Hodge/Helmholtz decomposition

3. Incompressible Euler equations
    1. The vorticity-stream formulation
    2. solution by energy method
    3. The particle-trajectory method
    4. The search for singular solutions

4. Incompressible Navier-Stokes equations
    1. weak solutions, existence
    2. strong solutions, uniqueness and regularity
    3. mild solutions
    4. inviscid limit and boundary layer

# References

We will mostly cover selected sections from the following.

1. Vorticity and Incompressible Flow, by Majda and Bertozzi.
2. Lectures on Navier-Stokes equations, by Tsai

Files of these books, other references, and scans of my lecture notes will be available in a Nextcloud webfolder, whose link will be given to the audience.

# Evaluation

The evaluation is based on homework assignments and class participation.

There will be 5 assignments. The assignments and their solutions will be posted in the Nextcloud webfolder.

# Instructor and lectures

**Instructor:** Dr. Tai-Peng Tsai, Math building room 109, phone 604-822-2591, ttsai at math.ubc.ca.

**Lectures:** Mon Wed Fri 10-10:50am in MATH 105.

Because there are 11 registered students, it does not seem easy to change the time. Our first meeting is Monday Jan 9. Please email me if you cannot attend it although you are interested.

**Office hours:** Mon 16pm-17:15pm, Tue 14pm-15:15pm, and by appointment (Tsai's schedule).