

# Outline of Math 426, Introduction to Topology

Ben Williams

First Winter Term, 2022

## 1 Contact information

The instructor for this course is me, Ben Williams. I may be reached at [tbjw@math.ubc.ca](mailto:tbjw@math.ubc.ca).

The course website is <http://www.math.ubc.ca/~tbjw/426/index.html>.

**Office hours** will be set later, and posted on the website.

## 2 Meeting Times

The course meets Mondays, Wednesdays and Fridays at 12pm, in GEOG 242. A traditional lecture format will be followed.

## 3 Prerequisites

The formal prerequisites for this course are MATH 321, a second course in real analysis, and Math 322, a first course in group theory. At a minimum, you should be comfortable with arguments regarding continuity of functions from  $\mathbf{R}^n \rightarrow \mathbf{R}^m$  and about convergence of sequences. Some homework assignments will also assume knowledge of other kinds of mathematics—please let me know if some assignment requires some knowledge you do not have.

## 4 Purpose

This course feeds into Math 427/527, which is a first course in algebraic topology. You should learn the basic ideas of point-set topology, and the very beginnings of algebraic topology, namely, the theory of the fundamental group and covering spaces. I will emphasize the aspects of the theory that do not overlap with functional analysis. You should also develop a high level of fluency with abstract arguments in mathematics, and learn the fundamentals of category theory.

## 5 Textbook

There is no required textbook for this course. Munkres' *Topology* is recommended (a paperback version may be purchased for under \$60, as is Hatcher's *Algebraic Topology* (free online) and for some light reading *Counterexamples in Topology* by Steen and Seebach (about \$20). Lecture notes will be made available.

## 6 Homework

Homework will be assigned throughout the course, at a rate of one assignment of three questions every week. The first assignment will be due on Monday 19 September, at 11:59pm. Subsequent homework assignments will be due on Mondays as well, (with a day's extension for the assignment due on the week of Thanksgiving, 11 October 2021). If you can, submit homework on Canvas, in the format of a pdf compiled from  $\LaTeX$  (or from  $\TeX$  or another  $\TeX$  derivative). If this poses some difficulty for you, let me know. There will be 12 assignments, of which only your highest-scoring 10 will count.

In each homework assignment, a portion of the marks (up to 20%) will be given for exposition. In order to get full marks for this portion, your answers should be clear, easy to read, and not excessively wordy.

Warning: the last homework will be due on 5 December 2022.

## 7 Exams

There will be a midterm in class on Wednesday 19 October and a final exam.

## 8 Overall Course Grade

The overall course grade will be assigned based on homework (30%), the midterm (20%) and the final exams (50%).

## 9 List of Topics

The following is a list of topics that will be covered.

1. Topological spaces
2. Separation axioms
3. Continuous functions
4. Generating topologies
5. Induced topologies
6. Co-induced topologies
7. Closure, interior, boundary
8. Density
9. Sequence methods
10. Compactness
11. Compactifications
12. Connectedness and path connectedness

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| 13. Categories, functors and natural transformations | 19. Covering spaces  |
| 14. Groupoids  | 20. The relationship between the fundamental group and covering spaces |
| 15. Homotopy   | 21. Deck transformations   |
| 16. The homotopy category                            | 22. Topological groups and group actions (time permitting)             |
| 17. The fundamental groupoid                         | 23. Further topics as time permits                                     |
| 18. The Van Kampen Theorem                           |  |

## 10 Academic Integrity statement

The UBC policies relating to Academic Misconduct apply: <http://www.calendar.ubc.ca/vancouver/index.cfm?tree=3,54,111,959>.

For homework assignments, you are encouraged to discuss the problems with other students, but you must write your own solutions. It is prohibited to plagiarize someone else's work. It is also prohibited to post Math 426 homework questions on math-discussion websites before the due date of the homework assignment in question, and I will check.

## 11 Policy statement

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence.

UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom.

UBC provides appropriate accommodation for students with disabilities and for religious observances.

UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available at <https://senate.ubc.ca/policies-resources-support-student-success>.