MATH_V 605D D_201 2025W2 Topics in Applied Mathematics - Topics in Applied Mathematics

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Applied Stochastic Analysis

Tues/Thurs 2-3:30, Math 204

PIMS course zoom link: https://ubc.zoom.us/j/61272126742?pwd=sBn1dB8upMkDOM9jtlg24rhUZiLXg0.1 (https://ubc.zoom.us/j/61272126742?pwd=sBn1dB8upMkDOM9jtlg24rhUZiLXg0.1)

Instructor:
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Office: Math 112

Office Hours: TBD

Description: This course will introduce the major tools in stochastic analysis from an applied mathematics perspective. Topics to be covered include Markov chains (both discrete and continuous), Gaussian processes, Itô calculus, stochastic differential equations (SDEs), numerical algorithms for solving SDEs, forward and backward Kolmogorov equations and their applications. It will develop both theoretical tools and simulation methods, while also paying attention to physical principles and applications. The class will attempt to strike a balance between rigour and heuristic arguments: it will assume that students have some familiarity with analysis, but will generally avoid measure theory. The target audience is graduate students in applied mathematics or related fields, who wish to use these tools in their research for modelling or simulation.

The course will be divided roughly into two parts: the first part will focus on stochastic processes, particularly Markov chains, and the second part will focus on stochastic differential equations and their associated PDEs.

Prerequisites: Good upper-level undergraduate or early graduate knowledge of: probability, linear algebra, PDEs, and ODEs. Some prior experience with numerical analysis is helpful but not necessary.

Homework

Homework will be a critical part of the course. Lectures will mostly be theory, and examples, extensions, and applications will be assigned as homework problems. You must do these if you want to learn something from the course.

Many homework assignments will involve some computing. We will use Pluto notebooks from the high-level computing language Julia for the examples. You can use Julia or Python for submitted code, and you can submit code using a Jupyter or Pluto notebook.

There will be weekly homework assignments, generally due Friday at 11:59pm. Late homework will be penalized 5% per day, and will be accepted for up to 7 days. The lowest 2 homework scores will be dropped.

Homework is found in the "Assignments" folder of Canvas. Links are in the table below. Hand in homework by uploading a pdf of your solutions, and a .jl file or .ipynb with any code, to Canvas.

You are *strongly encouraged to work with others*, on the homework problems and to study. However, you must *write up your own solutions*. Solutions that are identical, or nearly so, will be considered as plagiarism and will be treated accordingly. The best way to ensure this doesn't happen is, (once you have discussed the problems with others), to find a place on your own to sit and write your solutions, away from the input of others. It is also a good practice to acknowledge the students you worked with, eg at the top of your assignment.

Homework solutions: They are in "Files/Solutions". Passwords are

hwN!!!

where N is the homework number. For example, for HW2, the password is hw2!!!

Grading Scheme

Homework 50%
Midterm 15%
Final Exam 35%

References

The lectures will follow these notes quite closely, at a pace of roughly one chapter per week:

- M. Holmes-Cerfon, *Applied Stochastic Analysis* **asabook.pdf (https://canvas.ubc.ca/courses/181078/files/43455601?wrap=1)** **(https://canvas.ubc.ca/courses/181078/files/43455601/download?download_frd=1)** .
  This has been published here: **https://bookstore.ams.org/cln-33** **(https://bookstore.ams.org/**

cln-33)
Hence, *please don't distribute the pdf!*

Some other references that are highly recommended include:

- W. E, T. Li, E. Vanden-Eijnden. *Applied Stochastic Analysis.*
  The book that is the most similar to our course.
- G. A. Pavliotis. *Stochastic Processes and Applications*.
  This is a good references for the second part of the course, on stochastic calculus and associated PDEs. You can access a free pdf via the UBC library.
- G. Grimmett and D. Stirzaker, *Probability and Random Processes.*
  Great reference for probability theory and Markov chains.
- C. Gardiner, *Stochastic Methods: A Handbook for the Natural and Social Sciences*.
  A fantastic reference to have on the shelf. Easy to look up formulas, lots of physical applications.

Other good references include:

- L. Koralov and Y. G. Sinai, *Theory of Probability and Random Processes* (Springer)
  More rigorous/theoretical construction of stochastic processes.
- B. Oksendal, *Stochastic Differential Equations* (Springer)
  Excellent introduction to stochastic calculus.
- I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus* (Springer)
  More rigorous/theoretical construction of Brownian motions and its various properties.
- R. Durrett, *Essentials of Stochastic Processes* (Springer)
  Highly accessible reference on Markov chains and martingales.
- R. Durrett, *Stochastic Calculus: A Practical Introduction*
  A more theoretical construction than we will see, but is a useful and accessible text.
- P. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations* (Springer)
  This is the bible on numerically solving SDEs.

Draft Schedule (updated as we go)

Week	**Topics**	**Readings & Materials**
January 6, 8	probability review; intro to Markov chains	Appendix 0.1, 0.2, Chapter 2 (2.1, 2.2)

January 13, 15	Markov chains (I): discrete-time Markov chains	Chapter 2
January 20, 22	Markov chains (II): detailed balance, MCMC	Chapter 3
Jan 27, 29	Continuous-time Markov chains	Chapter 4
February 3, 5	Gaussian processes & stationary processes	Chapter 5
February 10, 12	Brownian motion	Chapter 6
February 16-20	Spring break	
February 24, 26	midterm *(tentative Feb 26),* catchup class	
March 3, 5	Stochastic integration	Chapter 7
March 10, 12	Stochastic differential equations	Chapter 8
March 17, 19	Numerically solving SDEs	Chapter 9
March 24, 26	Forward & Backward equations for SDEs	Chapter 10
March 31, Apr 2	Some applications of the backward equation	Chapter 11
April 7, 9	Detailed balance, symmetry, eigenfunction expansions	Chapter 12

Policies

Weather Contingency Plan

You should check ubc.ca often during bad weather or snow. If a class session is cancelled, class will be held online. Instructions will be posted on Canvas. For those unable to participate in the online class, we will provide a recording on Canvas. If a cancellation impacts a midterm, we will reschedule to another class time.

University policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the UBC Senate website.

Course Summary:

Date	Details	Due