Course Information

Course Title	Course Code Number	Credit Value
Topics in Geometry	Math 602D	3

Reference Book:

A Course in Minimal Surfaces by T. Colding and W. Minicozzi

Prerequisites

Some background in differential geometry (MATH 525 and 526) is recommended.

Contact

Section	Course Instructor	Contact Details
201	Ailana Fraser	afraser@math.ubc.ca

Office hours will be posted on the MATH 602D Canvas page.

Course Structure

The instructional format for the course will consist of 2.5 hours of lectures per week at the scheduled class times, Tuesday and Thursday 11am-12:20pm in MATH 204. Students are expected to participate in the regularly scheduled lectures.

Course Description

Geometric variational problems have been studied by mathematicians for more than two centuries. The theory of minimal surfaces, for instance, was initiated by Lagrange in 1760. Minimization principles have been extremely useful in the solution of various questions in analysis, geometry, and topology.

Minimal surfaces are defined as surfaces that locally minimize area subject to given boundary conditions, and geometrically, are characterized by having zero mean curvature. The geometry of the ambient manifold influences the behaviour of minimal surfaces through a second order effect, and minimal submanifold methods have proven to have powerful and far reaching applications related to the interconnections between the geometry and topology of manifolds and in geometry,

PDE and relativity. For example, minimal hypersurfaces are a fundamental tool in the study of positive scalar curvature in geometry and general relativity. More recently, and remarkably, minimax principles and unstable critical points have given us new tools and played a key role in the solution of old problems such as in the proof of the Poincaré Conjecture (2006), the proof of the Willmore Conjecture (2012), and the proof of Yau's Conjecture (2018).

This course will offer an introduction to minimal submanifolds and related variational problems. Topics covered include: first and second variation of volume, Morse index and stability, Plateau problem and existence theory for minimal surfaces, applications to curvature and topology, eigenvalue problems, harmonic maps, min-max theory and recent developments.

Schedule of Topics

This is an approximate week-by-week outline of topics for the course, subject to change.

Week	Topics	Sections
1	Preliminaries: review of geometry of submanifolds, introduction to minimal surfaces, examples	Ch 1 §1-2
2	First variation of volume and consequences	Ch 1 §3
3	Monotonicity formula, Bernstein theorem	Ch 1 §5
4	Weierstrass representation, maximum principles	Ch 1 §6, 7
5	Second variation of volume, Morse index and stability	Ch 1 §8
6	Minimal hypersurface stability, stable Bernstein theorem	Ch 2
7	Minimal cones, eigenvalue characterization of minimal surfaces in spheres	Ch 2
8	Positive curvature obstructing stability, index questions	references
9	Plateau problem	Ch 4 §1-3
10	Harmonic maps, ε-regularity	Ch 4 §5
11	Sacks-Uhlenbeck existence theory for minimal 2-spheres	Ch 4 §6
12	Applications to curvature and topology	references
13	Other topics, like min-max theory and recent developments	references

Learning Outcomes

By the end of the course, students will be able to define, explain, and give examples of the main concepts of the course, such as a definition and examples of minimal surfaces, and how variational methods can be used to study interconnections between the geometry and topology of manifolds.

Learning Activities and Assessments of Learning

Course mark will be based on class attendance and participation.

Learning Materials

- There will be 2.5 hours of lectures per week. Course information, materials and resources can be found on the MATH 602D Canvas page.

Academic Integrity

The academic enterprise is founded on honesty, civility, and integrity. As members of this enterprise, all students are expected to know, understand, and follow the codes of conduct regarding academic integrity. At the most basic level, this means submitting only original work done by you and acknowledging all sources of information or ideas and attributing them to others as required. This also means you should not cheat, copy, or mislead others about what is your work. Violations of academic integrity (i.e., misconduct) lead to the breakdown of the academic enterprise, and therefore serious consequences arise and harsh sanctions are imposed. Be sure you understand UBC's expectations: see the UBC Calendar entries on "Academic Honesty", "Academic Misconduct", and "Disciplinary Measures", and the Student Declaration and Responsibility.

University Policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the UBC Senate website.

Weather Contingency Plan

You should check ubc.ca often during bad weather or snow. If in-person activities are cancelled due to weather or other environmental conditions, class may be held online, and the Zoom link will be posted on Canvas. Please check the MATH 602D Canvas page for information.

Learning Analytics

This course will be using the following learning technologies: Canvas, Zoom. Many of these tools capture data about your activity and provide information that can be used to improve the quality of teaching and learning. In this course, we plan to use analytics data to:

- View overall class progress
- Track your progress in order to provide you with personalized feedback
- Review statistics on course content being accessed to support improvements in the course
- Track participation in discussion forums
- Assess your participation in the course

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