

Compressed Sensing (Math 555)

We develop tools and concepts from high-dimensional probability that underlie the mathematical theory of compressed sensing and related problems in data science. Not all results will be proven; proofs are selected for insight and useful techniques.

Evaluation is based on a (group) mini research project and presentation.

This is a tentative outline. We may not cover all material below.

0. Compressed sensing and the restricted isometry property

1. Behaviour of sums of random variables, non-asymptotic deviation inequalities

- a. Sub-Gaussian random variables
- b. Sub-exponential random variables
- c. Bernstein inequality

2. Concentration of measure

- a. Concentration on the sphere and in Gaussian space
- b. Implication: Lipschitz functions concentrate around their median
- c. Application: Johnson-Lindenstrauss lemma

3. Non-asymptotic random matrix theory and extrema of stochastic processes

- a. Connection of random matrices to stochastic processes
- b. Covering arguments
- c. Slepian inequality, Gordon inequality
- d. Matrix Bernstein inequality
- e. Application: Covariance estimation
- f. Advanced covering arguments: Dudley inequality, Generic chaining
- g. Application in compressed sensing: Restricted isometry property for sub-sampled Fourier matrices
- h. Majorizing measures theorem
- i. Conditioning of a random matrix restricted to a fixed set, with applications in generalized compressed sensing, including compressed sensing with neural nets
- j. Fano's inequality (without information theory), showing optimality of compressed sensing results

4. More applications

- a. Matrix completion
- b. Generalization error bounds in machine learning