Algebraic Geometry I

Welcome to Math 532, Algebraic Geometry I.

MWF 13:00-14:00 **Auditorium Annex 142 (https://wd10.myworkday.com/ubc/d/inst/15$337656/216$37312.htmld)**

Contact:

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Office hour: TBA

Text:

We will loosely follow parts of lecture notes by A. Gathmann, available **here (https://agag-gathmann.math.rptu.de/de/alggeom.php)**

We will also draw on Hartshorne's legendary textbook, which you can access via the UBC library (full text available online).

Piazza:

Use Piazza to discuss math questions, if you like.

Assessment:

There will be occasional homework assignments. You will be asked to submit peer evaluations of the homework submitted by your fellow students. The peer evaluations are an important part of your homework. Your course grade will be based on your homework, as well as on the peer evaluations you submit about your fellow students' homework.

Goal:

This is a one-semester introductory course in algebraic geometry. The goal is to prepare students to take more advanced courses in the subject (such as 533 or a topics course), which could eventually lead to research in the algebraic geometry.

Algebraic Geometry:

In a nutshell, algebraic geometry is the study of geometry by algebraic means.

For example, conic sections are described by their equations, such as $x^2 + y^2 = 1$. (This is a quadratic equation). Of course, we also consider higher order equations, such as cubics, $y^2 = x^3 + x + 1$. One of the most basic theorems in algebraic geometry is Bezout's theorem (18th century): the number of intersection points of two such algebraic curves in the plane, one of degree m and one of degree n is always nm. (For example, our above two curves should have 6 intersection points). There are many issues to be resolved, though, to make this always true. For example, some of the intersection points might have imaginary coordinates (this problem is resolved by working over the complex numbers). Or they might lie ”at infinity”. This is why we study ”projective” geometry. (By the way, from the point of view of complex projective geometry all conic sections, ellipses, parabolas, hyperbolas look the same.) Finally, the curves might be tangent to each other, which means that such intersection points have ‘multiplicity’. (They may even coincide!)

An interesting fact about cubic curves such as $y^2 = x^3 + x + 1$, is that their points form an abelian group. This group has applications in elliptic curve cryptography, for example.

As you can see, algebraic geometry is a very old subject, with a long history. But it is also a subject in which research is very active these days. Just recently (within the last 15 years), old questions such as ‘how many (parameterizable) curves of degree d in the plane pass through 3d-1 given points?’ have found answers, by the discovery of completely unexpected connections with quantum physics.

Algebraic geometry is also very important in number theory. The study of Diophantine equations (such as the solution of Fermat's last theorem) is impossible without algebraic geometry. The above mentioned elliptic curves feature prominently in this proof.

A lot of modern mathematics uses algebraic geometry as a basic tool. There is number theory (arithmetic geometry), topology and homotopy theory, or high energy theoretical physics, to name a few. A basic course in algebraic geometry is therefore recommended for all students serious about research in mathematics.