MATH_V 448A A_101 2026W1 Directed Studies in Mathematics - Directed Studies in Mathematics

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Math 448: Introduction to Stochastic Calculus

Description: This course will provide an introduction to the elements of stochastic calculus: Brownian motion, Itô integral, Itô's formula, stochastic differential equations (SDEs), numerically solving SDEs. It will show applications of these theories, including in some or all of mathematical finance, generative modelling, epidemiology, and physical sciences, depending on the interests of the class. There will be regular assignments and assessments, including an emphasis on computations, with a final assessment taking the form of a project or a final exam, to be determined in discussion with the class.

Prerequisites: A basic course in probability theory (e.g. Math 302, Math 318, or similar), along with a good mastery of this topic.

Date*:* Tuesday/Thursday 11-12:20
Location: MATH 204

Course Material

Links to course materials will be posted here as developed.

- **Review Sheet (https://canvas.ubc.ca/courses/195702/files/47958538?location=course_syllabus_195702&wrap=1)** ⭳ **(https://canvas.ubc.ca/courses/195702/files/47958538/download?location=course_syllabus_195702&download_frd=1)**

Learning Goals

- basic properties of a Brownian motion from its definition, and explain the informal relation $= dt$
- Explain the difference between the Itô integral and the Riemann integral
- Correctly apply the Itô formula
- Explain the key parts of a stochastic differential equation (SDE), e.g. drift term, diffusion term, Brownian motion/white noise, and explain what it means to be a solution to the SDE
- Solve some common SDEs analytically (Ornstein-Uhlenbeck process, Geometric Brownian Motion), and extract the solution's qualitative behaviour
- Simulate solutions to stochastic differential equations numerically, and identify ways to measure

the accuracy of the numerical solutions
- Use simulations to measure statistics associated with solutions to SDEs, including expectations, probabilities, and hitting times, and measure the accuracy of these statistical estimators
- Identify partial differential equations (PDEs) that can be solved to obtain statistics of solutions to SDEs, including the Black-Scholes equation

Instructors

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  - *Office hours:* TBD
- TBD (TA)
  - *Email*: TBD

Resources

There is no required textbook for this course, but below I will list two recommended books from which recommended readings will be assigned. Neither of these is perfectly tailored to the course, but both of these approximately follow its content.

- "**An Introduction to the Numerical Simulation of Stochastic Differential Equations** (https://epubs.siam.org/doi/book/10.1137/1.9781611976434) ", by D. Higham and P. Kloeden.
  - A pdf is available for download at the UBC Library (follow **this link** (https://gw2jh3xr2c.search.serialssolutions.com/?ctx_ver=Z39.88-2004&ctx_enc=info%3Aofi%2Fenc%3AUTF-8&rfr_id=info%3Asid%2Fsummon.serialssolutions.com&rft_val_fmt=info%3Aofi%2Ffmt%3Akev%3+J&rft.au=Kloeden%2C+Peter+E.&rft.date=2021-01-01&rft.pub=Society+for+Industrial+and+Applied+Mathematics&rft.isbn=9781611976427&rft_id=info

or search for the **title at the UBC Library** (https://ubc.summon.serialssolutions.com/ch?check=true&s.q=an+introduction+to+the+Numerical+Simulation+of+Stochastic+Differential+E ch?&ho=t&include.ft.matches=t&l=en&q=an%20introduction%20to%20the%20Numerical%20Sim , and then click the little triangle next to SIAM e-books, and click "link to book". Or try **this link** (https://epubs-siam-org.eu1.proxy.openathens.net/doi/book/10.1137/1.9781611976434) . Let the instructor know if you have trouble.)
  - This is a short, lively, succinct introduction to the ideas of stochastic calculus. It doesn't go into all the mathematical detail but it has plenty of references for those wishing to go further. It covers much of the basic material of this course including an introduction to stochastic calculus and stochastic differential equations, though it has more emphasis on numerical

methods than our course.

- "**Applied Stochastic Analysis** (https://bookstore.ams.org/view?ProductCode=CLN/33)", by Miranda Holmes-Cerfon.
  - A pdf is **here** (https://canvas.ubc.ca/courses/195702/files/47959180?location=course_syllabus_195702&wrap=1) (https://canvas.ubc.ca/courses/195702/files/47959180/download?location=course_syllabus_195702&download_frd=1) (please don't distribute)
  - This is based off of a set of lecture notes I wrote for a graduate course I teach of the name. Our course will follow the same order of presentation of ideas, starting with Brownian Motion. Be aware that this book is aimed at graduate students hence assumes more mathematical background than is assumed for our course; nevertheless it may be a good reference for those looking to go into further depth in some topics. Also be aware that this book avoids measure theory hence is not the most rigorous way to present the ideas.

In addition I will post (draft) lecture notes, ideally ahead of each lecture. These notes will be a template and will not have complete calculations; you can fill in the details during lectures.

Some additional references include:

- G. Grimmett and D. Stirzaker, *Probability and Random Processes.*
  Great reference for probability theory and Markov chains and Brownian motion.
- C. Gardiner, *Stochastic Methods: A Handbook for the Natural and Social Sciences*.
  A fantastic reference to have on the shelf. Easy to look up formulas, lots of physical applications.
- B. Oksendal, *Stochastic Differential Equations* (Springer)
  Excellent introduction to stochastic calculus.
- I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus* (Springer)
  More rigorous/theoretical construction of Brownian motions and its various properties. Considered a classic reference on the topic, but it is terse and hard to read.
- ett, *Essentials of Stochastic Processes* (Springer)
  ccessible reference on Markov chains and martingales.
- ett, *Stochastic Calculus: A Practical Introduction*
  theoretical construction than we will see, but is a useful and accessible text.

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Homework philosophy. Homework is a critical part of learning mathematics. Just as you don't get stronger from watching people lift weights, you also don't learn mathematics by watching someone (even your instructor, and especially even AI), solve problems in front of you. Learning math involves rewiring your brain cells, and you *must* go through the process of thinking through and solving problems yourself. Sometimes this means struggling with a problem for a long time and thinking about it from multiple angles, until you finally gain some understanding. It is not a process with

shortcuts.

Homework pragmatics. This class will have written homework assignments approximately once per week (about 6-8 assignments total). Homework can be handwritten or typed in LaTeX. Homework will be graded for completion, provided the responses look like reasonable attempts at each question.* Late homework will be accepted up until the solutions are posted (which will be a random amount of time after the homework is due, so you can work out the expectation of receiving the grade!) In addition, at the beginning of each homework you may indicate up to 3 questions (in order of preference) where you would like detailed feedback on your response. The grader will take time to provide feedback on at least some of these questions, within the time constraints of grading. This feedback is for your learning benefit only and will not count toward your grade.

* If the submitted homework is not a reasonable attempt to answer the questions given, or if it looks like it might be output from an LLM, then the instructor will discuss with the submittor and reserves the right to alter the grade based on this conversation.

Collaboration policy. *It is important that you do the homework assignments yourself* -- you may not submit work which is identical to another students'; nor may you submit output from AI. You are *encouraged,* however, to *work with other students and to share ideas while working*; if you do so *be sure to write up solutions yourself*. While AI is another possible study aid (provided you work with it as you would another student; see AI policy below), it is not recommended because it is too hard to avoid cognitive off-loading, and because you will not learn as deeply as you will if you think things through yourself and/or discuss with other people.

Computational Assignments

The course will feature ~3 computational assignments. These will take place in-class and will involve implementing algorithms to simulate stochastic processes, as well as using these simulations to explore properties of the stochastic processes. Assignments will be conducted in pre-assigned g the programming language of Python. Attendance is mandatory.

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wo options for assessments, which we will discuss and vote on as a class. Here is a brief of what they would involve, and some tentative thoughts on how they might be carried out.

Option A

- Midterm assessment
  - likely 15-minute oral assessment with instructor/TA based on homework questions, though I'm exploring the possibility of using the CBTF to have 2-3 low-stakes assessments instead
- Final project.
  - Would involve a written report (~10 pages) and oral assessment/discussion with instructor&TA,

both forming part of the final grading rubric
- Would also involve a short presentation to the class, mainly to celebrate learning, and graded mainly for participation.
- I'd reserve the last 1.5-2 weeks of class for working on projects and communicating certain project-relevant information
- You would work in groups of 2
- You would choose the topic yourself, so it is a chance to delve into a topic of interest more deeply. A reasonable project would be based on a textbook chapter or research paper, and would reproduce a figure or two and then adapt the model to a slightly new setting of your choosing. You would be expected to demonstrate mastery of a good selection of ideas from the class.

Option B

- Midterm assessments
  - Likely 2 midterm exams in-class
- Final Exam
  - Held during the final exam period
  - With this option, I'd cover a little bit more material during the semester (likely one more application area, e.g. backward SDEs from generative AI)

Grading (tentative)

Option A

Homework (completion)	30%
Midterm Assessment	10%
Computational Assignments	15%
ct	40%
n	5%

Option B

Homework (completion)	30%
Midterm Assessments	20%
Computational Assignments	15%
Final Exam	30%
Participation	5%

GenAI Policy

Generative artificial intelligence (GenAI) includes tools such as ChatGPT, Gemini, Copilot, Claude, etc. Students may use GenAI as a study aid and search tool, in the same way they may collaborate with other students or search the internet. However:

- Students must submit their own work. Do not submit any text, code or images created by GenAI. Do not submit any work that is paraphrasing output created by GenAI.
- Do not cite GenAI or its output as a source. Find primary sources such as published books and peer-reviewed research articles and cite those sources properly.

Violating the GenAI policy is academic misconduct. See **UBC Academic Integrity (https://academicintegrity.ubc.ca/chatgpt-faq/)** and **UBC Academic Misconduct (https://vancouver.calendar.ubc.ca/campus-wide-policies-and-regulations/student-conduct-and-discipline/discipline-academic-misconduct/3-academic-misconduct-ubc-students)** for more information.

See **UBC Guidance on Learning with GenAI (https://canvas.ubc.ca/http://UBC%20Guidance%20on%20Learning%20with%20GenAI)** for more information, and ask your instructors if you
have further questions

Policies

Weather Contingency Plan

You should check ubc.ca often during bad weather or snow. If a class session is cancelled, class will b ne. Instructions will be posted on Canvas. For those unable to participate in the online c ill provide a recording on Canvas. If a cancellation impacts an exam or in-class activity, v hedule to another class time.

policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions.

Details of the policies and how to access support are available on the UBC Senate website.

Course Summary:

Date	Details	Due
Wed Sep 16, 2026	**Homework 0 - Entry Survey (https://canvas.ubc.ca/courses/195702/assignments/2559228)**	due by 11:59pm