Course Information

Course Title	Course Code Number	Credit Value
Classical Differential Geometry	Math 424	3

Textbook:

- *Differential Geometry: A First Course in Curves and Surfaces* by T. Shifrin. Free online textbook.
- *Differential Geometry of Curves and Surfaces* by M. do Carmo. Optional reference textbook. This is not a free textbook, but is available online.

Prerequisites

Either (a) a score of 68% or higher in MATH 223 or (b) a score of 80% or higher in one of MATH 152, MATH 221; and either (a) a score of 68% or higher in MATH 227 or (b) a score of 80% or higher in one of MATH 217, MATH 254, MATH 264, MATH 317.

Contact

Section	Course Instructor	Contact Details
201	Ailana Fraser	afraser@math.ubc.ca

Office hours will be posted on the MATH 424 Canvas page.

Course Structure

The instructional format for the course will consist of 2.5 hours of synchronous lectures per week at the scheduled class times Tuesday & Thursday 2-3:20 in MATH 204. Students are expected to participate in the regularly scheduled synchronous lectures. There will be ample homework problems to help you practice technical skills and also develop understanding of the main concepts.

Schedule of Topics

This is an approximate week-by-week outline of topics for the course (subject to change).

Week	Topics	Shifrin Sections
1	Parametrized curves, regular curves, arc length, curvature, torsion	1.1, 1.2
2	Frenet equations, signed curvature of plane curves, local canonical form, fundamental theorem of curves	1.2, 1.3
3	Fundamental theorem of curves, global properties of curves, rotation index, surfaces in $\mathbb{R}^3$	1.3, 2.1
4	Regular surfaces, differential map (do Carmo 2.2, 2.3), first fundamental form	2.1
5	Gauss map and second fundamental form	2.2
6	Principal curvatures, Gauss and mean curvatures of surfaces	2.2
7	Surfaces of revolution, ruled surfaces, minimal surfaces	2.2
8	Intrinsic geometry of surfaces (do Carmo 4.2), Gauss and Codazzi equations	2.3
9	Theorema Egregium of Gauss, fundamental theorem of surfaces, covariant derivative, parallel transport	2.3, 2.4
10	Geodesics	2.4
11	Holonomy, preliminaries for Gauss-Bonnet theorem	3.1
12	Global Gauss-Bonnet Theorem and applications (do Carmo 4.5)	3.1

Course Description and Learning Outcomes

Differential geometry studies the properties of curves, surfaces, and higher-dimensional curved spaces using tools from calculus and linear algebra. A central concept in differential geometry is curvature. It is used to describe geometric features of objects such as a race track or the universe. In this course, we will study the curvature of curves, surfaces and higher dimensional spaces. We will study the mean curvature and Gauss curvature of surfaces, geodesics and parallel transport, Gauss' Theorema Egregium, and the Gauss-Bonnet Theorem.

This course will be of interest to any student in pure or applied mathematics. Differential geometry is the natural language for modern theoretical physics; in particular, for general relativity and superstring theory. In computer science, computer graphics and computer-aided geometric design draw on ideas from differential geometry. Differential geometry is used in digital signal processing, computer vision, and image processing. Finally, differential geometry also has applications in control theory, which may be of interest to students in engineering.

The main goal of the course is to develop an understanding of the fundamental concepts of the differential geometry of curves and surfaces and the skills necessary for its applications. Upon completion of the course students should be able to:

- Understand the curvature and torsion of a space curve, how to compute them, and how they suffice to determine the shape of the curve.
- Understand the definition of a smooth surface, and the means by which many examples may be constructed.
- Understand the various different types of curvature associated to a surface, and how to compute

them.
- Understand the first and second fundamental forms of a surface, how to compute them, and how they suffice to determine the local shape of the surface.
- Appreciate the distinction between intrinsic and extrinsic aspects of surface geometry.
- Understand the Gauss-Bonnet Theorem and be able to apply it.

Learning Activities and Assessments of Learning

Course mark will be based on homework/quizzes (40%), two midterms exams (30% each). There will be no final exam.

Homework/Quizzes

There will be weekly homework assignments which must be accessed from the MATH 424 Canvas page. The main goal of homework is to help you learn the material. You may discuss problems with classmates, but must complete the problems and yourself and write up your own solutions. Copying solutions from another student, from the web or from any other source, and turning them in as your own is a violation of the Academic Code. On the day the homework is due, there may be a short written quiz in class on one or two problems from the homework, or very similar to homework problems, which will be graded instead of the homework those weeks.

Midterms

There will be two midterms, the dates of which will be announced at the start of the term and posted on the MATH 424 Canvas page. The midterms will be during scheduled class time in MATH 204. Please be aware of Student Conduct during Examinations.

Specific details and policies for the midterms will be posted on Canvas.

Concessions

Students with concessions (e.g. for medical emergencies) will be required to take an alternate assessment (or might have the weight of the missed assessment transferred to other assessments). You can receive **one** concession during the term by submitting the Student Declaration of Academic Concession for Math Courses form to your instructor. Further concessions need to be discussed with the Academic Advisors of your Faculty. There cannot be any exception to this university-wide policy.

Learning Materials

There will be 2.5 hours of lectures per week. Course information, materials and resources can be found on the MATH 424 Canvas page.

Academic Integrity

The academic enterprise is founded on honesty, civility, and integrity. As members of this enterprise, all students are expected to know, understand, and follow the codes of conduct regarding academic integrity. At the most basic level, this means submitting only original work done by you and acknowledging all sources of information or ideas and attributing them to others as required. This also means you should not cheat, copy, or mislead others about what is your work. Violations of academic integrity (i.e., misconduct) lead to the breakdown of the academic enterprise, and therefore serious consequences arise and harsh sanctions are imposed. Be sure you understand UBC's expectations: see the UBC Calendar entries on "Academic Honesty", "Academic Misconduct", and "Disciplinary Measures", and the Student Declaration and Responsibility.

University Policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the UBC Senate website.

Weather Contingency Plan

Weather Contingency Plan for Class Sessions, Homework, Quizzes, and Exams: You should check ubc.ca often during bad weather or snow. If in-person activities are cancelled due to weather or other environmental conditions, class may be held online, and the Zoom link will be posted on Canvas. If homework is due, you may be required to submit it on Canvas. Please check the MATH 424 Canvas page for information. If a cancellation impacts a midterm exam or quiz, we will reschedule to another class time. If you are registered to write exams at the Centre for Accessibility, I encourage you to contact your CFA advisor to discuss the weather contingency plan for this course.

Learning Analytics

This course will be using the following learning technologies: Canvas, Crowdmark, Zoom. Many of these tools capture data about your activity and provide information that can be used to

improve the quality of teaching and learning. In this course, we plan to use analytics data to:

- View overall class progress
- Track your progress in order to provide you with personalized feedback
- Review statistics on course content being accessed to support improvements in the course
- Track participation in discussion forums
- Assess your participation in the course

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